Non-Exact Trig Equations

Solve for θ over [0°, 360°]

Ex 1) $\sin \theta = \dfrac{2}{3}$

must use positive value

$\theta = \sin^{-1}\left(\dfrac{2}{3}\right)$

$\theta_r = 41.810°$

$\theta = 41.810°, 138.190°$

$Q_1 \quad \theta = \theta_r$

$Q_2 \quad \theta = 180° - \theta_r$

$Q_3 \quad \theta = 180° + \theta_r$

$Q_4 \quad \theta = 360° - \theta_r$

Ex 2) $2\cos\theta + 1 = \dfrac{1}{3}$

$\cos\theta = -\dfrac{1}{3}$

$\theta_r = \cos^{-1}\left(\dfrac{1}{3}\right)$

$\theta_r = 70.529°$

$\theta = 109.471°, 250.529°$

Solve for θ over $[0, 2\pi]$

Ex 3) $\sin \theta = 0.6$

$\theta = \sin^{-1}(0.6)$

$\theta_r = 0.644$

$\theta = 0.644, 2.498$

$Q_1 \quad \theta = \theta_r$

$Q_2 \quad \theta = \pi - \theta_r$

$Q_3 \quad \theta = \pi + \theta_r$

$Q_4 \quad \theta = 2\pi - \theta_r$

Ex 4) $\dfrac{\cos\theta}{2} + 1 = \dfrac{3}{5}$

$\dfrac{\cos\theta}{2} = -\dfrac{2}{5}$

$\cos\theta = -\dfrac{4}{5}$

$\theta_r = \cos^{-1}\left(\dfrac{4}{5}\right)$

$\theta_r = 0.644$

$\theta = 2.498, 3.785$

Give exact solutions where possible

Ex 5) $\tan^2\theta - 5\tan\theta + 4 = 0$ over $[0, 2\pi]$

$$(\tan\theta - 1)(\tan\theta - 4) = 0$$

$\tan\theta = 1$ $\tan\theta = 4$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ $\theta_r = \tan^{-1}(4)$

$\theta_r = 1.326$

$\theta = 1.326, 4.467$

Ex 6) $2\tan^2\theta + \tan\theta - 3 = 0$ $\theta \in \mathbb{R}$

$$(2\tan\theta + 3)(\tan\theta - 1) = 0$$

$\tan\theta = -\frac{3}{2}$ $\tan\theta = 1$

$\theta_r = \tan^{-1}(\frac{3}{2})$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\theta_r = 0.983$ $+2\pi k$ $+2\pi k$

$\theta = 2.159, 5.300$

$+2\pi k$ $+2\pi k$ $k \in \mathbb{Z}$

yellow WS
except lmn