Proving Identities

to show that two expressions are identical

Use these identities:

$\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

$x^2 + y^2 = r^2$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

all identities can be squared

for example:
$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

Some strategies to verify:

1) Work with the more complicated side first
2) As needed write everything in terms of $\sin\theta$, $\cos\theta$, $\tan\theta$
3) Replace more complicated expressions with simpler ones
4) Try to write all terms over same denominator

Prove the following identities for all permissible values of θ.

ex. $1 + \cot^2\theta = \csc^2\theta$

	Left-Hand Side	Right-Hand Side
Turn $\cot^2\theta$ into into $\cos^2\theta/\sin^2\theta$ because they are equal	$1 + \dfrac{\cos^2\theta}{\sin^2\theta}$	$\csc^2\theta$
Write 1 as $\sin^2\theta/\sin^2\theta$ to make a common denominator	$\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}$	
	$\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$	
Replace $\sin^2\theta + \cos^2\theta$ with 1 because they are equal	$\dfrac{1}{\sin^2\theta}$	
Write $1/\sin^2\theta$ as its reciprocal because they are equal	$\csc^2\theta$	

Use any of these to end the question. They all mean that the two sides are now proven to be the same.

$\therefore$ LHS = RHS

$\therefore$ QED

$\therefore \square$

ex. $\dfrac{2-\sin^2\theta}{\cos\theta} = \sec\theta + \cos\theta$

	Left-Hand Side	Right-Hand Side
Split up the numerator into two separate fractions with the same denominator	$\dfrac{2}{\cos\theta} - \dfrac{\sin^2\theta}{\cos\theta}$	$\dfrac{1}{\cos\theta} + \cos\theta$
Replace $\sin^2\theta$ with $1 - \cos^2\theta$ because they are equal	$\dfrac{2}{\cos\theta} - \dfrac{(1-\cos^2\theta)}{\cos\theta}$	
Split into two separate fractions again and cancel $\cos\theta$ on top and bottom	$\dfrac{2}{\cos\theta} - \dfrac{1}{\cos\theta} + \dfrac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}}$	
Subtract first two fractions by subtracting value in numerator	$\dfrac{1}{\cos\theta} + \cos\theta$	$\therefore \square$

ex. $2\sin^2\theta - 1 = \sin^2\theta - \cos^2\theta$

	Left-Hand Side	Right-Hand Side
Replace 1 with $\sin^2\theta + \cos^2\theta$ because they are equal	$2\sin^2\theta - (\sin^2\theta + \cos^2\theta)$	$\sin^2\theta - \cos^2\theta$
Distribute negative in bracket	$2\sin^2\theta - \sin^2\theta - \cos^2\theta$	
Subtract $\sin^2\theta$ terms	$\sin^2\theta - \cos^2\theta$	$\therefore \square$

<u>Option 2: A different method</u>

	Left-Hand	Right-Hand
Replace $\cos^2\theta$ with $1 - \sin^2\theta$ because they are equal	$2\sin^2\theta - 1$	$\sin^2\theta - (1-\sin^2\theta)$
Distribute negative in bracket		$\sin^2\theta - 1 + \sin^2\theta$
Add $\sin^2\theta$ terms		$2\sin^2\theta - 1$

$\therefore$ QED

Difference of Squares Identities

Recall: $a^2 - b^2 = (a + b)(a - b)$

Recall: $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = (1 - \sin\theta)(1 + \sin\theta)$

Likewise: $\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = (1 - \cos\theta)(1 + \cos\theta)$

ex. $\dfrac{\cos\theta}{1 - \sin\theta} = \sec\theta + \tan\theta$

	Left-Hand Side	Right-Hand Side
Multiply 1 - sin θ by its conjugate (1 + sinθ)	$\dfrac{\cos\theta}{(1-\sin\theta)}\dfrac{(1+\sin\theta)}{(1+\sin\theta)}$	$\sec\theta + \tan\theta$
	$\dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$	
Replace 1 - $\sin^2\theta$ with $\cos^2\theta$ because they are equal	$\dfrac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$	
Cancel cosθ	$\dfrac{1+\sin\theta}{\cos\theta}$	
Split into two separate fractions with same denominator	$\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$	
Change into different trig functions that are equal to 1/cosθ and sinθ/cosθ	$\sec\theta + \tan\theta$	

$\therefore LHS = RHS$