Reciprocal Trig Identities

$\tan^2\theta + 1 = \sec^2\theta$ $\qquad\qquad$ $1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta = \sec^2\theta - 1$ $\qquad\qquad$ $\cot^2\theta = \csc^2\theta - 1$

$= (\sec\theta - 1)(\sec\theta + 1)$ $\qquad\qquad$ $= (\csc\theta - 1)(\csc\theta + 1)$

ex. Prove $\dfrac{1 + \tan^2\theta}{\tan^2\theta} = \csc^2\theta$

	Left-Hand Side	Right-Hand Side
Replace $1 + \tan^2\theta$ with $\sec\theta$ because they are equal	$\dfrac{\sec^2\theta}{\tan^2\theta}$	$\csc^2\theta$
Replace $\sec^2\theta$ and $\tan^2\theta$ with equivalent trig ratios so they have a common denominator of $\cos^2\theta$	$\dfrac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$	
Divide fractions and cancel	$\dfrac{1}{\cancel{\cos^2\theta}} \cdot \dfrac{\cancel{\cos^2\theta}}{\sin^2\theta}$	
	$\csc^2\theta$	$\therefore$ QED

ex. $\dfrac{\sin^2 x}{\sec x + 1} = \cos x - \cos^2 x$

	Left-Hand Side	Right-Hand Side
Mulitply secx +1 by it's conjugate (secx - 1)	$\dfrac{\sin^2 x}{(\sec x + 1)(\sec x - 1)}(\sec x - 1)$	$\cos x - \cos^2 x$
	$\dfrac{\sin^2 x(\sec x - 1)}{\sec^2 x - 1}$	
Replace $\sec^2 x - 1$ with $\tan^2 x$ because they are equal	$\dfrac{\sin^2 x(\sec x - 1)}{\tan^2 x}$	
Turn $\tan^2 x$ into $\sin^2 x/\cos^2 x$ and divide fractions	$\dfrac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x}(\sec x - 1)$	
Change secx into reciprocal 1/cosx.	$\cos^2 x \cdot \sec x - \cos^2 x$	
Divide out cosx	$\cos^{\cancel{2}} x \cdot \dfrac{1}{\cancel{\cos x}} - \cos^2 x$	
	$\cos x - \cos^2 x$	$\therefore$ LHS=RHS

ex. $\dfrac{1 - \sec^2\theta}{\sin\theta\csc\theta} = -\tan^2\theta$

Left-Hand Side	Right-Hand Side
$\dfrac{1-\sec^2\theta}{\sin\theta \cdot \frac{1}{\sin\theta}}$	$-\tan^2\theta$
$1-(\tan^2\theta+1)$	
$\cancel{1}-\tan^2\theta\cancel{-1}$	
$-\tan^2\theta$	$\therefore \square$

Turn cscθ into its reciprocal 1/sinθ

Replace $\sec^2\theta$ with $\tan^2\theta$ because they are equal

Cancel +1 and -1

ex. $\dfrac{\sin\theta + \cos\theta}{\sec\theta + \csc\theta} = \dfrac{\sin\theta}{\sec\theta}$

Left-Hand Side	Right-Hand Side
$\dfrac{\sin\theta+\cos\theta}{\frac{(\sin\theta)}{(\sin\theta)}\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\frac{(\cos\theta)}{(\cos\theta)}}$	$\dfrac{\sin\theta}{\frac{1}{\cos\theta}}$
$\dfrac{\sin\theta+\cos\theta}{\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}}$	$\sin\theta\cdot\cos\theta$
$\cancel{\sin\theta+\cos\theta}\cdot\dfrac{\sin\theta\cos\theta}{\cancel{\sin\theta+\cos\theta}}$	
$\sin\theta\cos\theta$	$\therefore$ QED

Turn secθ and cscθ into their reciprocal functions

Multiply by opposite function to make a common denominator (sinθcosθ)

Divide fractions and cancel reciprocal