

Double Angle Identities

ex. If $\sin \theta = \frac{2}{3}$ determine $\sin(2\theta)$ if θ is in quad 1

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{2}{3} \right) \left(\frac{\sqrt{5}}{3} \right) \\ &= \frac{4\sqrt{5}}{9} \end{aligned}$$

$$\begin{aligned} x^2 &= r^2 - y^2 \\ &= 3^2 - 2^2 \\ x &= \sqrt{5} \end{aligned}$$

ex. If $\tan \theta = -\frac{2}{3}$ over $(\frac{3\pi}{2}, 2\pi)$, determine $\cos(2\theta)$

$$\begin{aligned} 3^2 + (-2)^2 &= r^2 \\ 9 + 4 &= r^2 \\ r &= \pm \sqrt{13} \end{aligned}$$

$$\begin{aligned} \cos(2\theta) &= 2 \cos^2 \theta - 1 \\ &= 2 \left(\frac{3}{\sqrt{13}} \right)^2 - 1 \\ &= 2 \left(\frac{9}{13} \right) - 1 \\ &= \frac{18}{13} - \frac{13}{13} \\ &= \frac{5}{13} \end{aligned}$$

Prove the identity below for all permissible values of θ :

$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$$

LHS	RHS
$\frac{1 - \tan^2 \theta}{\sec^2 \theta}$	$\cos^2 \theta - \sin^2 \theta$
$\frac{1}{\sec^2 \theta} - \frac{\tan^2 \theta}{\sec^2 \theta}$	
$\cos^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta}$	
$\frac{1}{\cos^2 \theta}$	
$\cos^2 \theta - \sin^2 \theta$	
	$\therefore \square$