Combinations

A set or collection of things in <u>no particular order</u>

In permutations: ORDER MATTERS
In combinations: Order does NOT matter

In general,
the number of combinations of "n"
things taken or chosen "r" at a time

$$\boxed{nCr = \frac{n!}{(n-r)!\,r!}}$$ Reads "**n choose r**"

Perms:		Combs:
Phone numbers	**VS.**	Card hands
Locker Combinations		Lottery Numbers
License Plates		Committees

Ex) In how many ways can a committee of 4 be chosen from 4 teachers and 3 students if

$_nC_r = \frac{n!}{(n-r)!\,r!}$

(a) all are equally eligible?

$$_7C_4 = \frac{7!}{(7-4)!\,4!} = \frac{7\cdot 6\cdot 5\cdot \cancel{4!}}{3!\,\cancel{4!}} = 35$$

(b) If Amanda, one of the students, has to be on the committee, how many are possible?

$$\underbrace{_1C_1}_{\text{Amanda}} \cdot {_6C_3} = 20$$

(c) the committee must include 2 teachers AND 2 students?

Teachers · Students

$$_4C_2 \cdot {_3C_2}$$

$$\frac{4!}{(4-2)!\,2!} \cdot \frac{3!}{(3-2)!\,2!}$$

$$\frac{4\cdot 3\cdot \cancel{2!}}{2!\,\cancel{2!}} \cdot \frac{3\cdot \cancel{2!}}{\cancel{2!}}$$

$$6 \cdot 3$$

$$18$$

Ex) A cribbage hand consists of 6 cards

52
13♡ 13◇ 13♧ 13♤

(a) How many different hands are possible?

$_{52}C_6$

(b) How many hands with 3 kings only are possible?

kings • Rest

$_4C_3 \cdot {}_{48}C_3$

(c) How many hands with 2 or more kings?

multiply

o1 2 kings $\quad {}_4C_2 \cdot {}_{48}C_4 = 1{,}167{,}480$

o2 3 kings $\quad {}_4C_3 \cdot {}_{48}C_3 = 69{,}184$

o3 4 kings $\quad {}_4C_4 \cdot {}_{48}C_2 = 1{,}128$

add

$1{,}237{,}792$

Ex) A class has 15 girls and 20 boys.

(a) How many committees of eight consisting of 3 girls and 5 boys can be made?

boys · girls

$_{20}C_5 \cdot {}_{15}C_3$

(b) If Jill, Melissa, and Adam must be on the committee of eight?

J+M $\quad$ A $\quad$ boys • girls

$_2C_2 \cdot {}_1C_1 \cdot {}_{19}C_4 \;\; {}_{13}C_1$

(c) A committee of 5 with at least 3 girls?

3 girls $\quad {}_{15}C_3 \cdot {}_{20}C_2 =$

+

4 girls $\quad {}_{15}C_4 \cdot {}_{20}C_1 =$

+

5 girls $\quad {}_{15}C_5 \cdot {}_{20}C_0 =$

Solving Equations with Combinations

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Ex) Solve for n

(a) $_nC_2 = 21$

$$\frac{n!}{(n-2)!\,2!} = 21$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\cdot 2} = 21$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$$n = 7 \quad \cancel{n = -6}$$

(b) $_nC_{n-2} = 6$

$$\frac{n!}{(n-(n-2))!\,(n-2)!} = 6$$

$$\frac{n!}{2!\,(n-2)!} = 6$$

$$\frac{n(n-1)\cancel{(n-2)!}}{2!\,\cancel{(n-2)!}} = 6$$

$$n(n-1) = 12$$

$$n^2 - n - 12 = 0$$

$$(n+3)(n-4) = 0$$

$$\cancel{n = -3} \quad n = 4$$

Ex) Solve for n if $2(_nC_2) = {}_{n+1}C_3$

$$\frac{\cancel{2}\,n!}{(n-2)!\,\cancel{2!}} = \frac{(n+1)!}{(n+1-3)!\,3!}$$

$$\frac{n!}{\cancel{(n-2)!}} = \frac{(n+1)!}{\cancel{(n-2)!}\,3!}$$

$$6 = \frac{(n+1)!}{n!}$$

$$6 = \frac{(n+1)\cancel{(n)!}}{\cancel{n!}}$$

$$n = 5$$

Quiz FRIDAY

① Pink WS
② p. 731 #15

Permutations only

→ Fundamental Counting Principle

→ Restrictions ☆ Fill first
i.e. Female President etc.
Start and end with vowel

→ Repeating Letters → divide by # of repeats!

→ Cases – multiply across & add down ↓
– Alternating boys + girls
at least / at most

→ Arrangements – 2 people won't sit together
Subtract Total – together