Binomial Theorem

Binomial Expansion - is the expansion (multiplying out) of a binomial according to its power

$(x + y)^0$ $\quad 1$

$(x + y)^1$ $\quad 1x + 1y$

$(x + y)^2$ $\quad 1x^2 + 2xy + 1y^2$

$(x + y)^3$ $\quad 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x + y)^4$ $\quad 1x^4 + \boxed{4}x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

-Sum of exponents of each term is equal to the power of binomial
-Number of terms is always 1 more than the power of the binomial
-Pattern of the exponents:
- 1st term of binomial goes down by 1
- 2nd term of binomial goes up by 1 in expansion

Pascal's Triangle

1	Row 0	${}_0C_0$
1 1	Row 1	${}_1C_0$ ${}_1C_1$
1 2 1	Row 2	${}_2C_0$ ${}_2C_1$ ${}_2C_2$
1 3 3 1	Row 3	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$
1 4 6 4 1	Row 4	${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$
1 5 10 10 5 1	Row 5	${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$
1 6 15 20 15 6 1	Row 6	${}_6C_0$ ${}_6C_1$ ${}_6C_2$ ${}_6C_3$ ${}_6C_4$ ${}_6C_5$ ${}_6C_6$

If given $(x + y)^n$ is there an easy way to:
a) write out the expansion?
b) find a given term?

Use the Binomial Theorem

$$(x + y)^n = {}_nC_0x^{n-0}y^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \dots + {}_nC_nx^{n-n}y^n$$

ex. Expand $(x + 2y)^4$

$$(x+2y)^4 = \cancel{{}_4C_0}^{1}\, x^4 \cancel{(2y)}^{0} + \cancel{{}_4C_1}^{4}\, x^3 (2y)^1$$

$$+ \cancel{{}_4C_2}^{6}\, x^2 (2y)^2 + \cancel{{}_4C_3}^{4}\, x^1 (2y)^3 + \cancel{{}_4C_4}^{1}\, \cancel{x}^{0} (2y)^4$$

$$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$$

${}_nC_0 = 1$
${}_nC_1 = n$
${}_nC_{n-1} = n$
${}_nC_n = 1$

ex. Expand $(\overset{x}{a^2} \overset{y}{- 2b})^3$

$$= \cancel{{}_3C_0}^{1}\, (a^2)^3 \cancel{(-2b)}^{0} + \cancel{{}_3C_1}^{3}\, (a^2)^2 (-2b)^1$$

$$+ \cancel{{}_3C_2}^{3}\, (a^2)^1 (-2b)^2 + \cancel{{}_3C_3}^{1}\, \cancel{(a^2)}^{0} (-2b)^3$$

$$= a^6 - 6a^4b + 12a^2b^2 - 8b^3$$

${}_nC_0 = 1$
${}_nC_1 = n$
${}_nC_{n-1} = n$
${}_nC_n = 1$

p. 742
#1, 4, 7, 8