

Finding a Particular Term

General form $t_{k+1} = {}_n C_k (x)^{n-k} (y)^k$

Determine the 9th term of $(x - 3)^{10}$

$$\begin{aligned} t_9 &= {}_{10} C_8 (x)^2 (-3)^8 \\ &= \binom{10}{8+1} = (45)(x^2)(6561) \\ &= 295245x^2 \end{aligned}$$

ex. Find the 6th term of $(\overset{x}{3x} - \overset{y}{2y})^8$ $k=5$

$$\begin{aligned} t_6 &= {}_8 C_5 (3x)^3 (-2y)^5 \\ &= 56 (27x^3) (-32y^5) \\ &= -48384x^3y^5 \end{aligned}$$

ex. Find the 3rd term of $(3x^4 - 1)^{9n}$

$$\begin{aligned}
 t_3 &= {}_9C_2 (3x^4)^7 \left(-\frac{1}{x^3}\right)^2 \\
 &= (36) (2187x^{28}) \left(\frac{1}{x^6}\right) \\
 &= 78732 x^{22}
 \end{aligned}$$

Finding a missing value

ex 1) Find the term that contains x^{12} of $(-x^3 + 2)^6$

option 1

$$t_{k+1} = {}_n C_k (x)^{n-k} (y)^k$$

$$x^{12} = {}_6 C_k (-x^3)^{6-k} (2)^k$$

$$x^{12} = (x)^{18-3k}$$

$$12 = 18 - 3k$$

$$-6 = -3k$$

$$k = 2$$

\therefore term 3

$$(x^3)^6, (x^3)^5, (x^3)^4, (x^3)^3, \dots$$

$$x^{18}, x^{15}, (x^{12}), x^9, \dots$$

The pattern decreasing by 3

\therefore term 3

ex 2) Find the term that contains x^{-3} of $(x + \frac{1}{3x})^7$

option 1:

$$x^{-3} = (x)^{7-k} \left(\frac{1}{x}\right)^k$$

$$x^{-3} = (x)^{7-k} (x)^{-k}$$

$$-3 = 7 - 2k$$

$$-10 = -2k$$

$$k = 5$$

\therefore term 6

option 2:

$$(x)^7 \left(\frac{1}{x}\right)^0, (x)^6 \left(\frac{1}{x}\right)^1, (x)^5 \left(\frac{1}{x}\right)^2$$

$$x^7, x^5, x^3, x^1, x^{-1}, x^{-3}, \dots$$

The pattern decreasing by 2
 \therefore term 6

ex 3) Find the term containing x^2 in the expansion of $(x^3 - \frac{a}{x})^{10}$

$$x^2 = (x^3)^{10-k} \left(\frac{1}{x}\right)^k$$

$$x^2 = x^{30-3k} x^{-k}$$

$$x^2 = x^{30-4k}$$

$$2 = 30 - 4k$$

$$-28 = -4k$$

$$k = 7$$

\therefore term 8

$$(x^3)^{10} \left(\frac{1}{x}\right)^0, (x^3)^9 \left(\frac{1}{x}\right)^1, (x^3)^8 \left(\frac{1}{x}\right)^2$$

$$x^{30}, x^{26}, x^{22}, x^{18}, x^{14}, \dots$$

The pattern is decreasing by 4
 \therefore term 8

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 #2, 5, 11
 p. 746 #9