Fractional Exponents and Radicals

$3^4 = 3 \times 3 \times 3 \times 3 = 81$
The 4 indicates repeated multiplication.

A rational exponent has no meaning for this format.
$3^{1/2}$ "cannot be done" as repeated multiplication.
But it can be done with a calculator.
$3^{1/2} = 1.732050808...$

x	$x^{\frac{1}{2}}$
1	1
4	2
9	3
16	4
25	5

x	$x^{\frac{1}{3}}$
1	1
8	2
27	3
64	4
125	5

$$\therefore x^{\frac{1}{2}} = \sqrt{x} \qquad \therefore x^{\frac{1}{3}} = \sqrt[3]{x}$$

In general...
When n is a natural number and x is a rational number,

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

Ex. 1 Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}}$ b) $0.25^{\frac{1}{2}}$ c) $(-8)^{\frac{1}{3}}$ d) $\left(\frac{16}{81}\right)^{\frac{1}{2}}$

a) $\sqrt[3]{1000}$ = 10

b) $\sqrt[2]{0.25}$ = 0.5

c) $\sqrt[3]{-8}$ = -2

d) $\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9}$

"**Bottom Out**"
Whatever the bottom value of a rational exponent, this is the value that goes out of the radical sign. The top value stays by the base.

Ex 2: $8^{\frac{2}{3}} = (\sqrt[3]{8})^2$ or $\sqrt[3]{8^2}$

When *m* and *n* are natural numbers, and *x* is a rational number, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

Ex 3: a) Write $26^{\frac{2}{5}}$ in radical form in two ways.

$(\sqrt[5]{26})^2$ $\sqrt[5]{26^2}$

b) Write $\sqrt[2]{6^5}$ and $(\sqrt[4]{19})^3$ in exponent form.

$6^{\frac{5}{2}}$ $19^{\frac{3}{4}}$

Ex 4: Evaluate. *Usually easier to take the root first!

$16^{\frac{3}{2}}$

$(\sqrt{16})^3$

$(4)^3$

64

$(-27)^{\frac{4}{3}}$

$(\sqrt[3]{-27})^4$

$(-3)^4$

81

$8^{\frac{5}{3}}$

$(\sqrt[3]{8})^5$

2^5

32

$\left(\frac{-64}{27}\right)^{\frac{2}{3}}$

$$\frac{(\sqrt[3]{-64})^2}{(\sqrt[3]{27})^2} = \frac{(-4)^2}{3^2}$$

$$= \frac{16}{9}$$