

Negative Exponents

$$\frac{1}{x'} = \frac{x^{-1}}{1} \quad \text{OR} \quad \frac{1}{x^{-1}} = x' \quad \text{They are reciprocals}$$

Ex) Simplify: Our goal is to not have any negative exponents!

$$\frac{x^{-3}}{1} = \frac{1}{x^3} \quad \frac{1}{x^{-4}} = x^4 \quad \frac{1}{2^{-2}} = 2^2 = 4$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} \quad 3^{-4} = \frac{1}{3^4} = \frac{1}{81} \quad \frac{1}{3^{-3}} = 3^3 = 27$$

* Always move the negative exponent first then solve.

Negatives and Rational exponents

Ex) Evaluate:

$$\begin{aligned} 4^{-\frac{1}{2}} &= \frac{1}{4^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{4}} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} (-8)^{\frac{1}{3}} &= \frac{1}{(-8)^{\frac{1}{3}}} \\ &= \frac{1}{\sqrt[3]{-8}} \\ &= \frac{1}{-2} \end{aligned}$$

$$\begin{aligned} 27^{-\frac{2}{3}} &= \frac{1}{27^{\frac{2}{3}}} \\ &= \frac{1}{(\sqrt[3]{27})^2} \\ &= \frac{1}{3^2} = \frac{1}{9} \end{aligned}$$

$$\begin{aligned} 16^{-\frac{3}{4}} &= \frac{1}{16^{\frac{3}{4}}} \\ &= \frac{1}{\sqrt[4]{16}^3} \\ &= \frac{1}{(2)^3} = \frac{1}{8} \end{aligned}$$

$$\begin{aligned} (-64)^{-\frac{2}{3}} &= \frac{1}{(-64)^{\frac{2}{3}}} \\ &= \frac{1}{(\sqrt[3]{-64})^2} \\ &= \frac{1}{(-4)^2} = \frac{1}{16} \end{aligned}$$

$$\begin{aligned} 4^{-\frac{3}{2}} &= \frac{1}{4^{\frac{3}{2}}} \\ &= \frac{1}{(\sqrt{4})^3} \\ &= \frac{1}{8} \end{aligned}$$

To write the reciprocal of a fraction,
switch the numerator and denominator.

ex) $\frac{2}{5}$ and $\frac{5}{2}$

Ex) $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3$
 $= \frac{(5)^3}{(2)^3}$
 $= \frac{125}{8}$

$\left(-\frac{4}{3}\right)^{-2} = \left(-\frac{3}{4}\right)^2$
 $= \frac{(-3)^2}{(4)^2}$
 $= \frac{9}{16}$

$\left(\frac{25}{49}\right)^{-\frac{1}{2}} = \left(\frac{49}{25}\right)^{\frac{1}{2}}$
 $= \frac{\sqrt{49}}{\sqrt{25}}$
 $= \frac{7}{5}$

$\left(\frac{16}{81}\right)^{-\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}}$
 $= \frac{(4\sqrt{81})^3}{(4\sqrt{16})^3}$
 $= \frac{3^3}{2^3} = \frac{27}{8}$