

Natural Logs

Recall, log is called a "common log" ie. \log_{10}

How about, $\log_e = \ln$ called the natural log

where $e = 2.71828\dots$

Euler

Note, $\ln e = 1$

$\log_e e = 1$

All log laws still apply

ex. $\ln(M \cdot N) = \ln M + \ln N$

Finally, works like log in solving eq'ns

ex. $e^{3x} = 4$ Could use log or ln

$$\begin{aligned} \log e^{3x} &= \log 4 \\ 3x \log e &= \log 4 \\ \frac{3x \log e}{3 \log e} &= \frac{\log 4}{3 \log e} \\ x &= 0.462\dots \end{aligned}$$

$$\begin{aligned} \ln e^{3x} &= \ln 4 \\ 3x \ln e &= \ln 4 \\ \frac{3x \ln e}{3} &= \frac{\ln 4}{3} \\ x &= 0.462\dots \end{aligned}$$

ex. $e^{\ln 2x} = 8$

$$\begin{aligned} \ln e^{\ln 2x} &= \ln 8 \\ (\ln 2x) \ln e &= \ln 8 \end{aligned}$$

$$\begin{aligned} 2x &= 8 \\ x &= 4 \end{aligned}$$

→ NEVER

$$\begin{aligned} \ln 2x &= \ln 8 \\ 0 &= \ln 8 - \ln 2x \\ 0 &= \ln \left(\frac{8}{2x} \right) \end{aligned}$$

$$\begin{aligned} e^0 &= \frac{8}{2x} \\ 1 &= \frac{8}{2x} \end{aligned}$$

$$\begin{aligned} 2x &= 8 \\ x &= 4 \end{aligned}$$

ex. $e^x = 3^{x+2}$

$$\ln e^x = \ln 3^{x+2}$$

$$x \ln e = (x+2) \ln 3$$

$$x = x \ln 3 + 2 \ln 3$$

$$x - x \ln 3 = 2 \ln 3$$

$$x(1 - \ln 3) = 2 \ln 3$$

$$x = \frac{2 \ln 3}{(1 - \ln 3)}$$

$$x = -22.281 \dots$$

ex. $\ln(x^2 - 1) - \ln(x + 1) = 1$

$$\ln \left(\frac{x^2 - 1}{x + 1} \right) = 1$$

$$\ln \left(\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} \right) = 1$$

$$\ln(x-1) = 1$$

$$e^1 = x - 1$$

$$\boxed{e + 1 = x} \leftarrow \text{non calc}$$

or $x = 3.718 \leftarrow \text{on calculator section}$

$\frac{1}{2}$ WS
7 questions