Continuous Growth + Decay Problems

Ex 1: The population of a certain country is 28 million and grows continuously at a rate of 3% annually. Determine how many years will it take for the population to reach 40 million.

$A = Pe^{rt}$ where P = population at the start
A = population at the end
r = growth rate (as a decimal)
t = time (in years)

$\frac{40}{28} = \frac{28}{28}e^{0.03t}$

$\frac{\ln\left(\frac{40}{28}\right)}{0.03} = \frac{0.03t}{0.03}\ln e$ (1)

$t = 11.889$ years

$\ln 40 = \ln 28 + 0.03t \ln e$

$\frac{\ln 40 - \ln 28}{0.03} = \frac{0.03t}{0.03}$

$t = 11.889$ years

Ex 2: A radioactive substance decays so that the amount present, P, in grams after t years is: $P = 50e^{-0.135t}$. Determine the half life of the substance.

Half life = the amount of time it takes for the substance to decay to half its original amount.

$25 = 50e^{-0.135t}$

$0.5 = e^{-0.135t}$

$\frac{\ln 0.5}{-0.135} = \frac{-0.135t \ln e}{-0.135}$

$5.134 = t$

years

Ex 3: A population of 600 bacteria will triple in 11 hours.

a) Determine the rate of growth.

b) Determine how many hours in will take for the initial population to double with the same rate of growth.

a) $A = Pe^{rt}$

$3 = 1e^{11r}$

$\frac{\ln 3}{11} = \frac{11r \ln e}{11}$

$r = 0.09987\ldots$

$r = 0.100$

$\frac{1800}{600} = \frac{600e^{11r}}{600}$

$3 = 1e^{11r}$

b) $2 = 1e^{\left(\frac{\ln 3}{11}\right)t}$

$\frac{\ln 2}{\left(\frac{\ln 3}{11}\right)} = \frac{\left(\frac{\ln 3}{11}\right)t \ln e}{\left(\frac{\ln 3}{11}\right)}$

$6.940 = t$

hours

Logarithmic Scale Problems

Ex 4: The most intense earthquake ever recorded was in Chile in May 1960, with magnitude 9.5. In January 2010, Haiti experienced an earthquake with magnitude 7.0. Determine how many times as intense the Chile earthquake was than the Haiti earthquake. Express your answer as a whole number.

$M = \log\left(\frac{I}{S}\right)$ where M = magnitude of the earthquake

I = intensity of the earthquake

S = intensity of the standard earthquake

Chile

$9.5 = \log\left(\frac{I}{S}\right)$

$10^{9.5} = \frac{I}{S}$

$10^{9.5} S = I$

Haiti

$7 = \log\left(\frac{I}{S}\right)$

$10^{7} = \frac{I}{S}$

$10^{7} S = I$

Comparsion $\frac{\text{Chile}}{\text{Haiti}}$

$I = \frac{10^{9.5} S}{10^{7} S}$

$= 10^{2.5}$

$= 316.227\ldots$

$= 316$

times more intense

p. 434

#4, 5, 9, 10, 11