Combining Transformations

In summary,

$y = f(x) \text{ ----> } y = af\ b(x - c) + d$

$(x, y) \text{ ----> } \left(\frac{x}{b} + c, ay + d\right)$

$$y = af\ b(x - c) + d$$

stretch/compress vertically "y"

stretch/compress horizontally "x"

horizontal shift left/right

vertical shift up/down

Combining Transformations

Ex) Describe $y = -3f\left(\frac{1}{2}(x - 4)\right) - 5$ in words

(a b c d)

vertical reflection over the x-axis
vertical stretch by a factor of 3
horizontal stretch by a factor of ½
translated right 4 units
translated down 5 units

Ex) The point (3, -4) is on the graph of y = f(x). Determine its corresponding point after the following transformations of f(x): $y = 3f\left(\frac{1}{2}(x - 4)\right) - 5$

original

$(x, y) \rightarrow \left(\frac{x}{b} \pm c, ay \pm d\right)$

$(3, -4) \rightarrow (2x + 4, 3y - 5)$

$(2(3) + 4, 3(-4) - 5)$

$(10, -17)$

Ex) The point (3, -4) is on the image graph after the following transformations of f(x): $y = 3f\left(\frac{1}{2}(x - 4)\right) - 5$. Determine its corresponding point on f(x).

original

$(x, y) \rightarrow (2x + 4, 3y - 5)$

$(3, -4)$

$2x + 4 = 3$ $\quad$ $3y - 5 = -4$

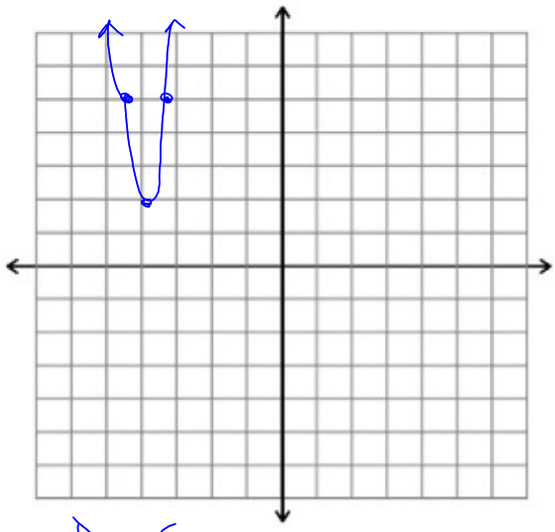
$x = -\frac{1}{2}$ $\quad$ $y = \frac{1}{3}$

$\left(-\frac{1}{2}, \frac{1}{3}\right)$

Ex) Sketch each function and state its domain and range.

a) $y = 3(2(x + 4))^2 + 2$

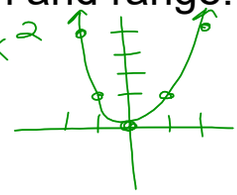
a b c d



D: $(-\infty, \infty)$

R: $[2, \infty)$

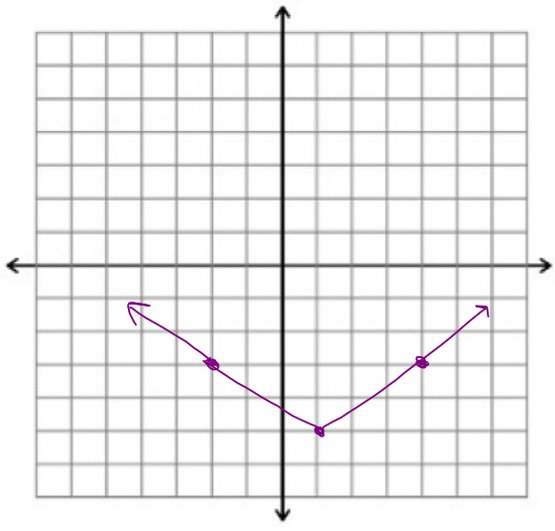
$y = x^2$



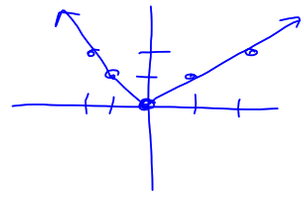
$(x, y) \rightarrow \left(\frac{x}{2} - 4, 3y + 2\right)$

$(-1, 1)$	$(-4.5, 5)$
$(0, 0)$	$(-4, 2)$
$(1, 1)$	$(-3.5, 5)$
$(2, 4)$	$(-3, 14)$

b) $y = 2\left|\frac{1}{3}(x - 1)\right| - 5$



$y = |x|$



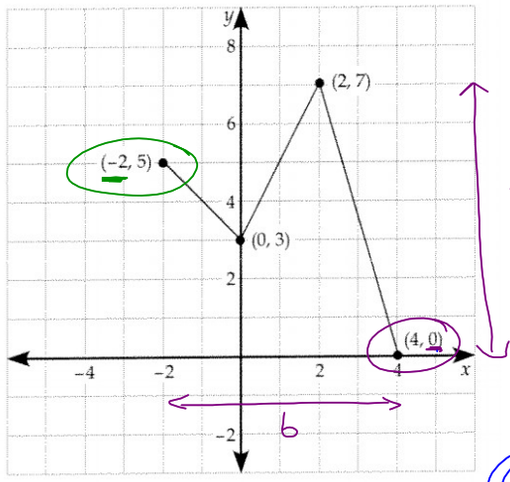
$(x, y) \rightarrow (3x + 1, 2y - 5)$

$(-1, 1)$	$(-2, -3)$
$(0, 0)$	$(1, -5)$
$(1, 1)$	$(4, -3)$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq -5\}$

Ex) The graph of f(x) is given below.

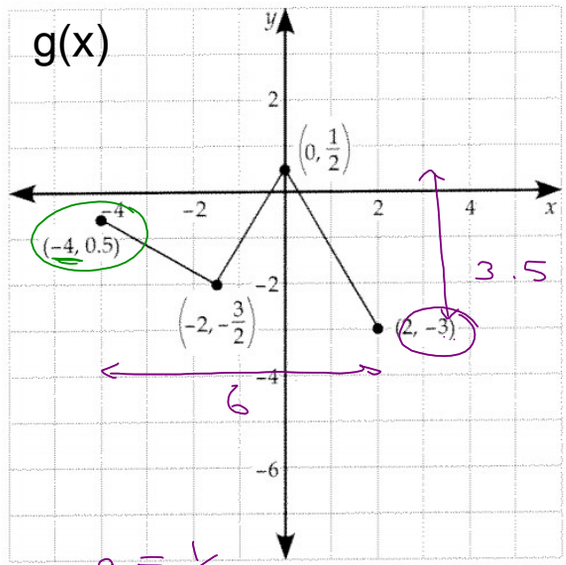


$\frac{0}{2} = 0$ $b = \frac{1}{2}$

(4, 0) $\xrightarrow{\times 2}$ (8, 0)

Write an equation for each new graph in terms of f(x).

a) g(x)



$a = \frac{1}{2}$

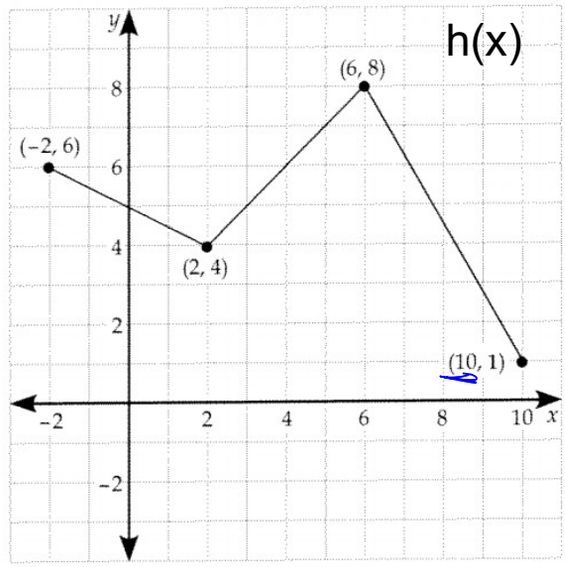
$b = 1$

$c = +2$

$d = -3$

$g(x) = \frac{1}{2} f(x + 2) - 3$

b) h(x)



$a = 1$

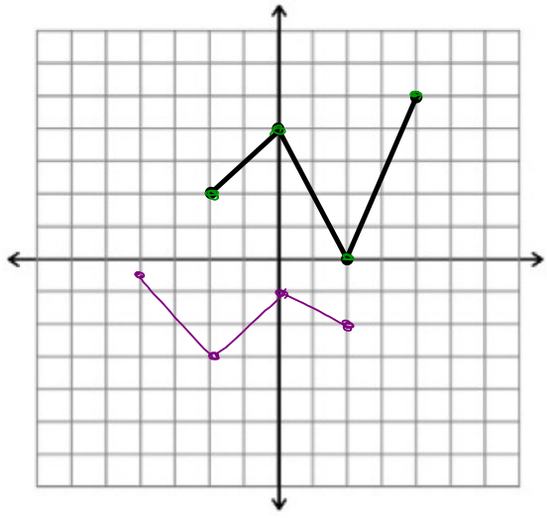
$b = \frac{1}{2}$

$c = -2$

$d = +1$

$h(x) = f\left(\frac{1}{2}(x - 2)\right) + 1$

Ex 1) Given T(x)

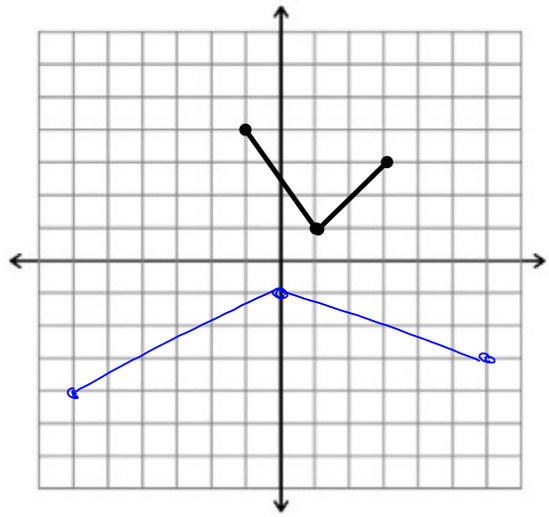


Graph: $m(x) = \frac{1}{2}T(-x) - 3$ (a, b, d)

$(x, y) \rightarrow (-x, \frac{y}{2} - 3)$

$(-2, 2)$	$(2, -2)$
$(0, 4)$	$(0, -1)$
$(2, 0)$	$(-2, -3)$
$(4, 5)$	$(-4, -\frac{1}{2})$

Ex 2) Given j(x)



Graph:

f(x) = -j(1/3x + 1)

$f(x) = \underset{a}{-}j(\underset{b}{\frac{1}{3}}(x \underset{c}{+3}))$

$(x, y) \rightarrow (3x - 3, -y)$

$(-1, 4)$	$(-6, -4)$
$(1, 1)$	$(0, -1)$
$(3, 3)$	$(6, -3)$

Extra Graphing ws

P. 226 #3-7, 9-11