

Feb 5

Logarithm Laws

Law 1: $\log_a(MN) = \log_a M + \log_a N$

ex. $\log_3(4 \cdot 5) = \log_3 4 + \log_3 5$
single log expanded log

Law 2: $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

ex. $\log_2\left(\frac{8}{3}\right) = \log_2 8 - \log_2 3$

Law 3: $\log_a M^P = P \cdot \log_a M$

ex. $\log_a 5^2 = 2 \log_a 5$

$\log_a(25) = 1 \log_a(5) + 1 \log_a(5)$

Extension:

ex. $\log \sqrt[3]{8} = \log 8^{\frac{1}{3}} = \frac{1}{3} \log 8$

Single Log \longrightarrow Expanded Log

Ex 1) $\log_k\left(\frac{L^7}{M}\right) = \log_k L^7 - \log_k M$
 $7 \log_k L - \log_k M$

Ex 2) $\log_k\left(\frac{L}{M}\right)^7 = 7 \log_k L - 7 \log_k M$

Ex 3) $\log_k\left(\frac{\sqrt{x^5}}{4\sqrt{x^3}}\right) = \log_k\left(\frac{x^{\frac{5}{2}}}{x^{\frac{3}{4}}}\right)$
 $= \frac{5}{2} \log_k x - \frac{3}{4} \log_k x$

$$\begin{aligned} \text{Ex 4) } \log_4\left(\frac{A}{xy^3}\right) &= \log_4 A - \log_4 xy^3 \\ &= \log_4 A - (\log_4 x + \log_4 y^3) \\ &= \log_4 A - \log_4 x - 3\log_4 y \end{aligned}$$

$$\begin{aligned} \text{Ex 5) } \log\left(\frac{4(x+1)}{z\sqrt{y}}\right) &= \log 4 + \log(x+1) - (\log z + \frac{1}{2}\log y) \\ \sqrt{y} &= y^{1/2} \\ &= \log 4 + \log(x+1) - \log z - \frac{1}{2}\log y \end{aligned}$$

Expanded Log \longrightarrow Single Log

$$\begin{aligned} \text{Ex 1) } \frac{1}{4}\log_5(x+1) - 3\log_5 2x &= \frac{\frac{1}{4}\log_5(x+1)}{3\log_5(2x)} \\ &= \log_5\left(\frac{(x+1)^{1/4}}{(2x)^3}\right) \end{aligned}$$

$$\begin{aligned} \text{Ex 2) } a\log 3 - \frac{1}{2}\log x - \log(a+1) \\ &= \log 3^a - \log x^{1/2} - \log(a+1) \\ &= \log\left(\frac{3^a}{\sqrt{x}(a+1)}\right) \end{aligned}$$

$$\begin{aligned} \text{Ex 3) } \log_7 x^2 + \log_7 y - \frac{5\log_7 x}{2} \\ &= \log_7 x^2 + \log_7 y - \frac{5}{2}\log_7 x \\ &= \log_7\left(\frac{x^2 y}{x^{5/2}}\right) \end{aligned}$$

$$\begin{aligned} \text{Ex 4) } \log_6 x + 3 \\ \text{(discuss tomorrow)} \end{aligned}$$

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1-3, 5, 8 abc,
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