Solving Log Equations

Feb 7

Solve for x

ex. $\log_{10}(x + 4) + \log_{10}5 = 2$

$\log_{10}((x+4)5) = 2$

$10^2 = 5x + 20$

$100 = 5x + 20$

$80 = 5x$

$x = 16$

Steps:

1) Rewrite as a single log

2) Rewrite as in exponential form

3) Solve for x

4) Check that values of x do not give a negative or zero argument → no solution

ex. $\log_2(x^2) - \log_2(x - 2) = 3$

$\log_2\left(\frac{x^2}{x-2}\right) = 3$

$(x-2)2^3 = \frac{x^2(x-2)}{x-2}$

$8x - 16 = x^2$

$0 = x^2 - 8x + 16$

$0 = (x-4)(x-4)$

$x = 4$

$\log_b N = x \rightarrow b^x = N$

Factor

$x^2 - 8x + 16$

$(x-4)(x-4)$

ex. $\log_3 x + \log_3(2x + 7) = 2$

$\log_3(x(2x+7)) = 2$

$3^2 = 2x^2 + 7x$

$0 = 2x^2 + 7x - 9$

$x = 1$ $\quad x = -\frac{9}{2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-7 \pm \sqrt{49 - 4(2)(-9)}}{2(2)}$

$= \frac{-7 \pm \sqrt{121}}{4}$

$= \frac{-7+11}{4} = 1$ $\quad = \frac{-7-11}{4} = -\frac{18}{4} = -\frac{9}{2}$

ex. $\log(8x + 4) = 1 + \log(x + 1)$

$\log(8x+4) - \log(x+1) = 1$

$\log\left(\frac{8x+4}{x+1}\right) = 1$

$10^1 = \frac{8x+4}{x+1}$

$10x+10 = 8x+4$

$2x = -6$

$x = -3$

No solution

ex. $2\log x - \log(x + 2) = \log(2x - 3)$

$\log\left(\frac{x^2}{x+2}\right) = \log(2x-3)$

$\frac{x^2}{x+2} = 2x-3$

$x^2 = (2x-3)(x+2)$

$0 = x^2 + x - 6$

$(x+3)(x-2) = 0$

$x = -3$ $\quad x = 2$

Factor

$2x^2+7x-9=0$

$x^2+7x-18=0$

$\left(x+\frac{9}{2}\right)\left(x-\frac{2}{2}\right)=0$

$(2x+9)(x-1)=0$

$x = -\frac{9}{2}$ $\quad x = 1$

ex. $\log_x 27 - \log_x 3 = 2\log_x y$

$\log_x\left(\frac{27}{3}\right) = \log_x y^2$

$9 = y^2$

$y = 3$ $\quad y = -3$

$y = 3$

p. 421

#1, 2, 4, 5, 9, 11, 13