Analyzing Rational Functions

Rational Functions = Functions of the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials and $q(x) \neq 0$.

$f(x) = \frac{x^2}{x-1}$, $x \neq 1$ $\quad$ $g(x) = \frac{2}{x^2-9}$, $x \neq \pm 3$

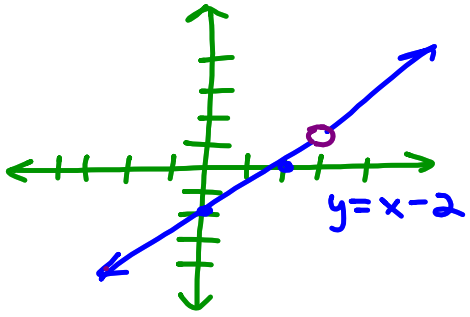
Holes vs. Vertical Asymptotes

Vertical asymptotes correspond to the non-permissible values in the equation of a function. Not all non-permissible values are vertical asymptotes. Some non-permissible values result in holes or points of discontinuity (if the numerator and denominator have common factors.)

ex. $f(x) = \frac{x^2 - 5x + 6}{x - 3}$

$= \frac{(x-3)(x-2)}{(x-3)}$

$= x - 2$, $x \neq 3$



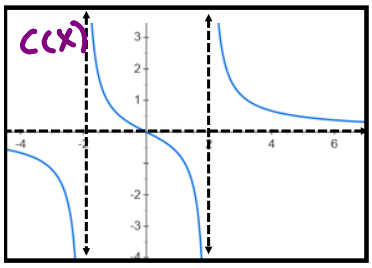
ex. Compare the following: $f(x) = \frac{x^2 - 2x}{4 - 2x}$ $\quad$ $g(x) = \frac{x^2 + 2x}{4 - 2x}$

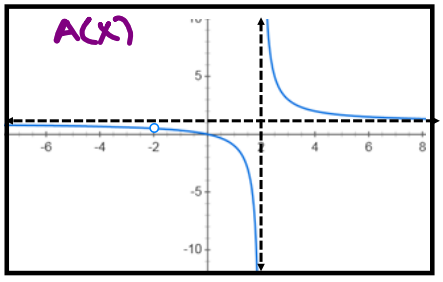
$(2-x)$
$(-x+2)$
$-(x-2)$

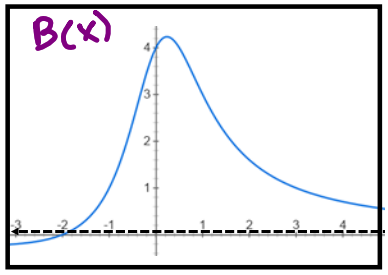
$f(x) = \frac{x(x-2)}{2(2-x)} = \frac{x(x-2)}{-2(x-2)} = -\frac{x}{2}$, $x \neq 2$ P.O.D.

$g(x) = \frac{x(x+2)}{2(2-x)} = \frac{x(x+2)}{-2(x-2)}$, $x \neq 2$ V.A.

ex. Match each graph with the equation of the rational function:







A(x) = $\frac{x^2 + 2x}{x^2 - 4}$

$= \frac{x(x+2)}{(x-2)(x+2)}$

$x \neq \pm 2$

B(x) = $\frac{2x + 4}{x^2 + 1}$

$= \frac{2(x+2)}{x^2+1}$

$x^2 + 1 \neq 0$

$x^2 \neq -1$

C(x) = $\frac{2x}{x^2 - 4}$

$= \frac{2x}{(x-2)(x+2)}$

$x \neq \pm 2$

2 options for **Horizontal Asymptotes:**

1) If the degree of the numerator is <u>less than</u> the degree of the denominator, the H.A. will be $y = 0$

$f(x) = \frac{1}{x-2}$ Deg 0 / Deg 1 $\qquad g(x) = \frac{x+2}{x^2-4}$ Deg 1 / Deg 2

2) If the degree is the <u>same</u> must look at the leading coefficients of the numerator and denominator.

$h(x) = \frac{2x^2}{x^2-1}$ Deg 2 / Deg 2

$y = \frac{2}{1}$

$y = 2$

$k(x) = \frac{x-3}{4-x}$

$y = \frac{1}{-1}$

$y = -1$

Ex. Determine the equations of all asymptotes and the coordinates of any points of discontinuity (if necessary).

$f(x) = \frac{x+1}{x^2+x-2}$

$= \frac{x+1}{(x+2)(x-1)}, \; x \neq -2, 1$

HA $y = 0$

VA $x = -2, 1$

$g(x) = \frac{x^2-3x+2}{1-x^2}$

$= \frac{(x-2)(x-1)}{-(x-1)(x+1)}$

$= \frac{(x-2)}{-(x+1)}, \; x \neq -1, 1$

HA $y = -1$

VA $x = -1$

P.o.d. $x = 1$ or $(1, \frac{1}{2})$

p. 115 #5, 6, 9adef, 10