Composite Functions

Refers to the combining of two functions f(x) and g(x) where the output of one function is used as the input of the other function.

Recall, $f(x) = 2x - 1$ find $f(3)$ ... $x = 3$

$f(3) = 2(3) - 1$
$= 5$

The notation used for compostion is

$\boxed{(f \circ g)(x) = f(g(x))}$

Inner brackets are done first.
First substitute into g, then into f.

Reads "f composed with g of x" or "f of g of x"

Ex. The tables below define two functions.
Use these tables to determine each value below.

x	f(x)
-2	8
-1	3
0	0
1	-1
2	0

x	g(x)
-2	3
-1	2
0	1
1	0
2	-1

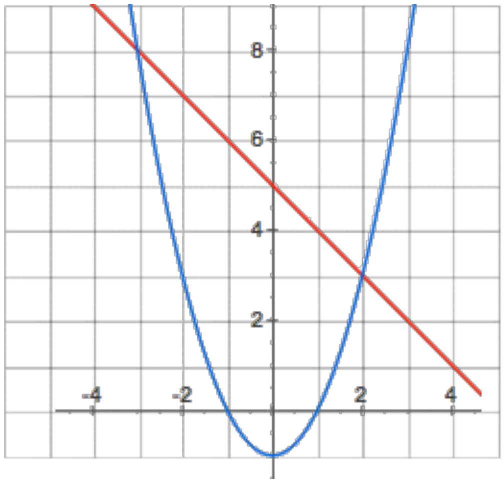
a) $f(g(-1))$
$f(2)$
0

c) $g(f(1))$
$g(-1)$
2

b) $f(f(2))$
$f(0)$
0

d) $g(g(2))$
$g(-1)$
2

Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below:



a) $f(g(-1))$

$f(0) = 5$

b) $g(f(3))$

$g(2) = 3$

c) $f(f(4))$

$f(1)$

4

Ex) If $f(x) = 4x$, $g(x) = x + 6$, and $h(x) = x^2$, find

a) $f(g(3))$

b) $g(h(-2))$

c) $h(h(2))$

a) $g(3) = 3+6$
$= 9$
$f(9) = 4(9)$
$= 36$

b) $g(h(-2)) = 10$

c) $h(2) = 2^2 = 4$
$h(h(2)) = 4^2$
$= 16$

Ex) If $f(x) = x^3 + 1$ and $g(x) = 2x$, find $(f \circ g)(x)$.

$(f \circ g)x$
$= f(g(x))$
$= f(2x)$
$= (2x)^3 + 1$
$= 8x^3 + 1$

Steps:

1) Write the expression for the function of g(2x) in the g(x) 'spot' in the composition

2) Now substitute this expression (2x) into function in the x 'spot'

3) Simplify (if necessary)

Ex) Given $f(x) = 5x$ and $g(x) = x^2 + 1$, find
a) $(f \circ g)(x)$
b) $(g \circ f)(x)$

a) $f(g(x))$
$f(x^2+1)$
$5(x^2+1)$
$5x^2+5$

b) $g(f(x))$
$g(5x)$
$(5x)^2+1$
$25x^2+1$

Notice that $(f \circ g)(x)$ and $(g \circ f)(x)$ do not necessarily have the same answer.

Ex) Given $f(x) = 4x$ and $g(x) = x^2 - x + 3$, find

a) $f(g(x))$

b) $g(f(x))$

c) $f(f(x))$

a) $f(g(x))$

$f(x^2 - x + 3)$

$4(x^2 - x + 3)$

$4x^2 - 4x + 12$

b) $g(f(x))$

$g(4x)$

$(4x)^2 - (4x) + 3$

$16x^2 - 4x + 3$

c) $f(f(x))$

$f(4x)$

$4(4x)$

$16x$

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