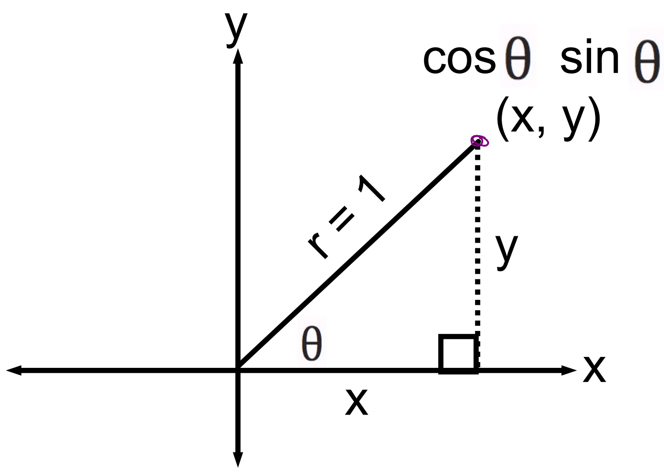
Coordinates of Points on Unit Circle



On unit circle **only**

$x^2 + y^2 = 1^2$

$\cos\theta = \dfrac{x}{r} = \dfrac{x}{1} = x$

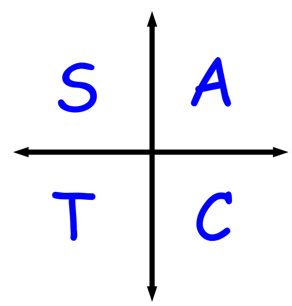
$\sin\theta = \dfrac{y}{r} = \dfrac{y}{1} = y$

Conclusion: For all points on the unit circle, since r = 1, we can replace $\cos\theta$ with x and $\sin\theta$ with y

SO, $P(\theta) = (\cos\theta, \sin\theta)$

How about, $\tan\theta = \dfrac{y}{x} = \dfrac{\sin\theta}{\cos\theta}$

Remember: Cast rule!



Ex. Is the point (2, 1/2) on the unit circle? Justify your answer.

$x^2 + y^2 = r^2$

$2^2 + \left(\frac{1}{2}\right)^2 = r^2$

$4 + \frac{1}{4} = r^2$

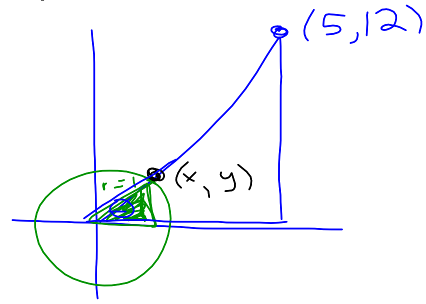
$\sqrt{\frac{17}{4}} = \sqrt{r^2}$

$\sqrt{\frac{17}{4}} = r$

$r \neq 1$ $\therefore$ the point $(2, \frac{1}{2})$ is not on the unit circle

Finding P(θ) on Unit Circle

Ex. If given a point with coordinates ($\overset{x}{5}$, $\overset{y}{12}$), determine the corresponding point on the unit circle.



$x^2 + y^2 = r^2$

$5^2 + 12^2 = r^2$

$25 + 144 = r^2$

$\sqrt{169} = r$

$13 = r$

$P(\theta) = (\cos\theta, \sin\theta)$

$P(\theta) = \left(\frac{x}{r}, \frac{y}{r}\right)$

$= \left(\frac{5}{13}, \frac{12}{13}\right)$

Ex.If $\cos\theta = -\frac{5}{15}$, find $\sin\theta$ given that θ is not in quadrant 2.

$\cos\theta = \frac{-5}{15} \quad \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

S A
T C

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = 15^2$

$25 + y^2 = 225$

$y^2 = 200$

$y = \pm\sqrt{200}$

$\sin\theta = \frac{-\sqrt{200}}{15}$

Ex. Given $\sin\theta = \frac{6}{\sqrt{61}} \frac{y}{r}$ and $\tan\theta < 0$, find $\cos\theta$.

S A
T C

$x^2 + 6^2 = (\sqrt{61})^2$

$x^2 = 61 - 36$

$x = \pm\sqrt{25}$

$x = \pm 5$

$\cos\theta = \frac{x}{r}$

$= \frac{-5}{\sqrt{61}}$

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3 New Reciprocal Trig Ratios or Functions

$$\text{secant}\,\theta = \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{x}{r}} = \frac{r}{x}$$

$$\text{cosecant}\,\theta = \csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{y}{r}} = \frac{r}{y}$$

$$\text{cotangent}\,\theta = \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$$

CAST still applies

S	A
T	C

Ex. Find $\cot\theta$ over $(\frac{\pi}{2}, \pi)$ if $\sin\theta = \frac{3}{5}$ $\begin{matrix} y \\ r \end{matrix}$

$\cot\theta = \frac{x}{y}$

$x^2 + 3^2 = 5^2$

$x^2 = 16$

$x = \pm 4$

$\cot\theta = \frac{-4}{3}$

$\pi/2$, π

S	A
T	C

p. 477
8, 10
p. 494
4, 13i, 14i

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