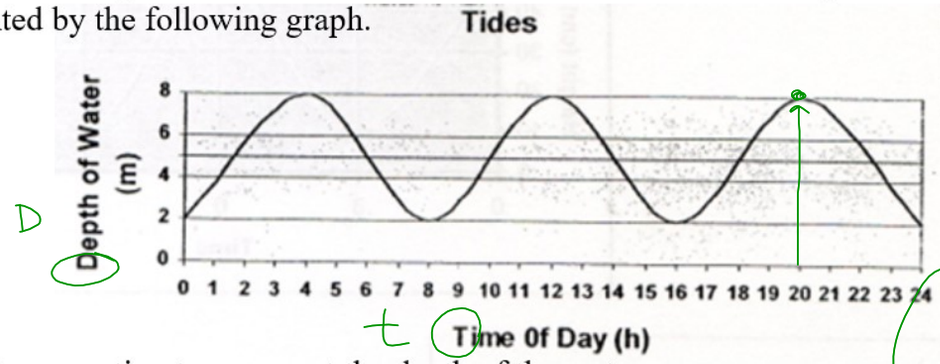
Modeling Real Life Situations using Trig Functions

1. The tides are the periodic rise and fall of water in the oceans. The depth of this water can be represented by the following graph.



$A = 3$

$B = \frac{2\pi}{8} = \frac{\pi}{4}$

$C =$

$D = 5$

a) Find an equation to represent the depth of the water.

b) How deep was the water at 8:00pm? → $20 = t$

c) If the water is 6.5m deep, what time of the day could it be?

no C values

cos → −3 A value

or cos → −4 C value

or sin → −2 C value

a) $D(t) = -3\cos\left(\frac{\pi}{4}t\right) + 5$

(other equations are possible)

b) $D(20) = -3\cos\left(\frac{\pi}{4}(20)\right) + 5$

$= -3(-1) + 5$

$= 8$ m

$\cos(5\pi)$

-4π

$\cos(\pi)$

-1

c) $6.5 = -3\cos\left(\frac{\pi}{4}t\right) + 5$

$-5 \quad\quad -5$

$\frac{1.5}{-3} = \frac{-3\cos\left(\frac{\pi}{4}t\right)}{-3}$

$-\frac{1}{2} = \cos\left(\frac{\pi}{4}t\right)$

$\frac{\cos^{-1}\left(-\frac{1}{2}\right)}{\left(\frac{\pi}{4}\right)} = \frac{\frac{\pi}{4}t}{\frac{\pi}{4}}$

$2.667 = t$

hours

Non-calc

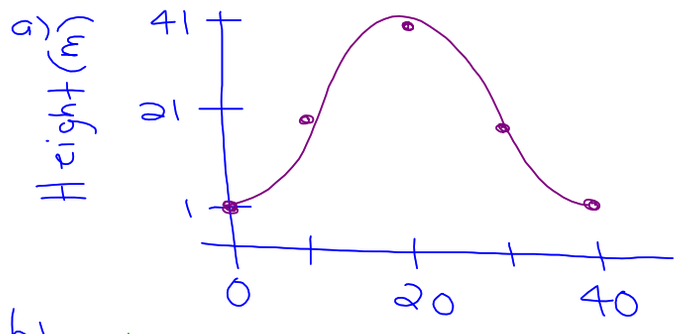
$\frac{2\pi}{3} = \frac{\pi}{4}t$

$\frac{2\pi}{3} \cdot \frac{4}{\pi} = t$

$\frac{8}{3}$ hrs $= t$

2. A Ferris wheel has a radius of 20m. It rotates once every 40s. Passengers get on at a point S, which is 1 m above ground level. Suppose you get on at S and the wheel starts to rotate.
   a) Sketch a graph of the function.
   b) Determine an equation for height in m as a function of time in sec.
   c) Calculate your height after 45 s. t

a)



$A = 20$

$B = \frac{2\pi}{40} = \frac{\pi}{20}$

$C = \sin \rightarrow -10$

$D = 21$

b) $h(t) = 20 \sin \frac{\pi}{20}(t-10) + 21$

c) $h(45) = 20 \sin\left(\frac{\pi}{20}(45-10)\right) + 21$

$= 20 \sin\left(\frac{35\pi}{20}\right) + 21$

$= 20 \sin\left(\frac{7\pi}{4}\right) + 21$

$= 20\left(-\frac{\sqrt{2}}{2}\right) + 21$

$= 6.858 \text{ m}$

$\sin\left(\frac{7\pi}{4}\right)$

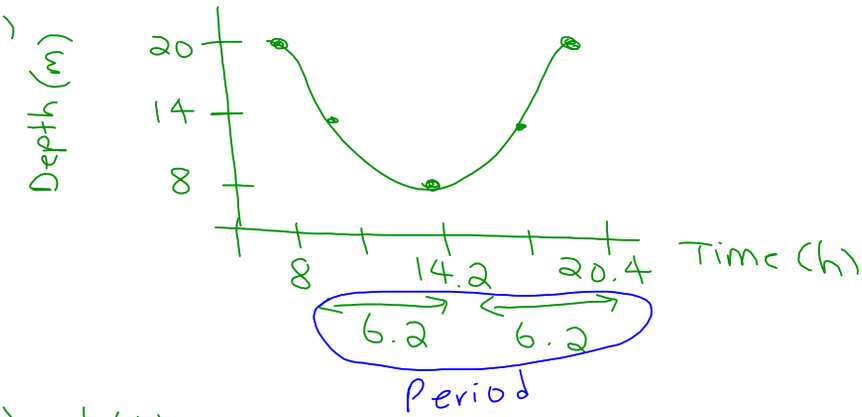
$-\frac{\sqrt{2}}{2}$

S | A

T | C

3. On a typical day at an ocean port, the water has a maximum depth of 20 m at 8:00am. The minimum depth of 8 m occurs as 6.2 h later. Assume the relation between the depth of the water and the time is a sinusoidal function.
   a) Sketch a graph of the function.
   b) Write an equation for the depth d of the water in metres at any time t in hours.
   c) Estimate one of the times when the water is 10 m deep.

a)



$A = 6$

$B = \frac{2\pi}{12.4} = \frac{\pi}{6.2}$

$C = \cos \rightarrow -8$

$D = 14$

b) $d(t) = 6\cos\frac{\pi}{6.2}(t-8) + 14$

c) $10 = 6\cos\frac{\pi}{6.2}(t-8) + 14$

$-14 \quad\quad -14$

$-\frac{4}{6} = \frac{6}{6}\cos\frac{\pi}{6.2}(t-8)$

$\frac{\cos^{-1}\left(-\frac{4}{6}\right)}{\left(\pi/6.2\right)} = \frac{\frac{\pi}{6.2}(t-8)}{\left(\pi/6.2\right)}$

$\left(\frac{\cos^{-1}\left(-\frac{4}{6}\right)}{\left(\pi/6.2\right)}\right) + 8 = t$

$12.540\ h = t$

Green WS
#1-3
(practice → NOT hand-in)