Factorial Notation!

n! reads "n factorial"

ex. 5! = $5 \times 4 \times 3 \times 2 \times 1 = 120$

ex. 8! = $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40\,320$

ex. n! = n(n - 1)(n - 2)(n - 3)...

$n \in \mathbb{N}$ ← is a natural number

How about 0! = 1

Likewise, n! = n(n - 1)!

or 8! = 8 · 7!

$8! = 8 \cdot 7 \cdot 6 \cdot 5!$

Simplify without a calculator:

a) $\frac{9!}{7!} = \frac{9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 72$

b) $\frac{8!}{5!\,3!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{3 \cdot 2 \cdot 1}} = 56$

c) $\frac{(n+4)!}{(n+2)!} = \frac{(n+4)(n+3)\cancel{(n+2)!}}{\cancel{(n+2)!}}$

$= n^2 + 7n + 12$

ex. Show $5! + 4! = 6 \cdot 4!$

$5 \cdot 4! + 1 \cdot 4!$ | $6 \cdot 4!$

$6 \cdot 4!$

$\therefore \square$

ex. Show $(k + 1)! + k! = (k + 2)k!$

$(k+1)k! + 1k!$ | $(k+2)k!$

$(k+1+1)k!$

$(k+2)k!$

$\therefore$ QED

ex. Show that $\dfrac{n(n-1)!(n-r+1)}{(n-r+1)!} = \dfrac{n!}{(n-r)!}$

$\dfrac{n(n-1)!(n-r+1)}{(n-r+1)(n-r)!}$ | $\dfrac{n!}{(n-r)!}$

$\dfrac{n!}{(n-r)!}$

$\therefore$ LHS = RHS