Permutations

An arrangement of all or part of a number of things in a definite order.

Order of the Arrangement Counts!!!

Permutations are used to generate:
- license plate numbers
- phone numbers
- social insurance numbers
- locker combinations

We show permutations with the symbol "$_nP_r$" which reads

the number of permutations of "n" things arranged or taken or picked "r" at a time

Ex)
(a) How many permutations can be formed from the letters in the word JUSTICE using all 7 letters?

$_7P_7$

In general, $\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$ $n \geq r$ $n \in N$

Using the above example..

option 1: $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

option 2: ${}_n^7P_r^7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7!$

option 3 : calculator

7 nPr 7 = 5040

(b) How many permutations can be formed from JUSTICE using only 5 letters at a time?

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

or ${}_7P_5$ says "using 7 objects *picking* or *arranging* them 5 at a time"

Also reads "**7 pick 5**"

On calculator ${}_7P_5 =$ [7] [nPr] [5] [=]

Or ${}_7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$

$= 2520$

Ex)
A group of 9 different books are to be selected and arranged on a shelf for display, picking 4 at a time. How many arrangements are possible?

$9 \cdot 8 \cdot 7 \cdot 6$

or

${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 3024$

Note: Arrangement, lineup, selected suggest order or perms.

Using ${}_nP_r$ to solve for n or r

Ex) Solve for n, given ${}_nP_2 = 30$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$

$$n^2 - n - 30 = 0$$

$$(n-6)(n+5) = 0$$

$$n = 6 \quad \cancel{n = -5}$$

Ex) Solve for r, given ${}_5P_r = 20$

$$\frac{5!}{(5-r)!} = 20$$

$$\frac{120}{(5-r)!} = 20$$

$$\frac{120}{20} = (5-r)!$$

$$6 = (5-r)!$$

$$6 = 3!$$

$$r = 2$$