Perms with Repeating Objects

Ex) How many ways can you arrange the letters of "WOW"?

$\_\_ \cdot \_\_ \cdot \_\_ = 6$ ways

W_1OW_2 WWO OWW $\frac{6}{2!} = 3$

W_2OW_1 WWO OWW

Since the W repeats twice, we correct by dividing by 2!

Ex) How many ways can you arrange the letters of WOWW?

$$\frac{4 \cdot 3 \cdot 2 \cdot 1}{3!} = 4$$

In general,
the number of perms of *n* objects
with *r* identical objects is $\boxed{\frac{n!}{r!}}$

Ex) Using all letters in the AARDVARK how many different arrangements are possible?

$$\frac{8!}{3! \cdot 2!}$$

A A R

$$\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2}$$

4

3360 ways

Perms with Restrictions

Note: Must ALWAYS fill restrictions first!!

Ex) Using the first 10 letters of the alphabet, how many different arrangements can you make if you must

A B C D E F G H I J

(a) start with a vowel

$\underset{V}{3} \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

(b) start and end with a vowel

$\underset{V}{3} \; 8 \; 7 \; 6 \; 5 \; 4 \; 3 \; 2 \; 1 \; \underset{V}{2}$

(c) **Not** start or end with a vowel

$\underset{C}{7} \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \; \underset{C}{6}$

Ex) Using all the letters of the word BRAINS (no reps), how many 4-letter arrangements are possible

(a) ending in "R"

$5 \cdot 4 \cdot 3 \cdot \underset{R}{1}$

(b) with consonants only

$\underset{C}{4} \cdot \underset{C}{3} \cdot \underset{C}{2} \cdot \underset{C}{1}$

(c) with consonants and vowels alternating

option 1: $\underset{C}{4} \cdot \underset{V}{2} \cdot \underset{C}{3} \cdot \underset{V}{1} = 24$

2: $\underset{V}{2} \cdot \underset{C}{4} \cdot \underset{V}{1} \cdot \underset{C}{3} = 24$

48

Ex) If there are six chairs in a row, how many ways can 3 boys and 3 girls sit if they must alternate?

$\frac{3}{B} \cdot \frac{3}{G} \cdot \frac{2}{B} \cdot \frac{2}{G} \cdot \frac{1}{B} \cdot \frac{1}{G} = 36 \times 2$

$= 72$

flip

Arrangements within Arrangements

Ex) How many ways can you arrange 8 people in a row if 3 people insist on sitting together?

$\boxed{3 \cdot 2 \cdot 1}\ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$6! \cdot 3!$

individuals + group: 5 — group only: 3

When items must be together, tie them together and count them as a single item.

There are now 6 items above, so arrange:

$6! \cdot 3!$

Ex) How many ways can you arrange 4 people in a row if 2 people must sit together?

$\boxed{2 \cdot 1}\ 2\ 1$

$3!\ 2!$

12 ways

Ex) How many ways can you arrange 3 Math books, 2 Physics books, and 5 English books on a shelf keeping the books of each subject area together?

$3! \cdot 3! \cdot 2! \cdot 5!$

Subjects: math, physics, english

$3 \cdot 2 \cdot 1$

$3 \cdot 2 \cdot 1 \quad | \quad 2 \cdot 1 \quad | \quad 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Ex) How many ways can 6 people sit in a row if 2 people must not sit together?

Total arrangements possible - total arrangements when together

$6! - \underbrace{5}_{2 \cdot 1} \; 4 \cdot 3 \cdot 2 \cdot 1$

$6! - 5! 2!$

$720 - 240$

480

Ex) How many different four-digit numbers can be made that are even using the digits 0, ~~1~~, ~~2~~, ~~3~~, 4, 5, 6 ?

option 1: $\underbrace{5}_{\text{not 0}} \; \underbrace{5}_{\substack{\text{add 0} \\ \text{back in}}} \; \underbrace{4}_{\substack{\text{even} \\ 2,4,6}} \; \underbrace{3}_{\text{even}} = 300$

option 2: $6 \cdot 5 \cdot 4 \; \underbrace{1}_{\substack{\text{even} \\ 0}} = 120$

420 numbers

Ex) How many different 3-digit numbers can be made using the digits 2, 3, 4, 5, and 0 if the number must be divisible by 5?

$\underset{\text{not } 0}{3} \cdot \underset{\neq 0}{3} \cdot \underset{5}{1} = 9$

$4 \cdot 3 \cdot \underset{0}{1} = 12$

21

Ex) How many different numbers of at least three digits can be formed from the integers 1, 2, 3, 4, 5? 3 or more

$5 \cdot 4 \cdot 3 = 60$

$5 \cdot 4 \cdot 3 \cdot 2 = 120$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

300

Pink WS
p. 701 # 2, 3, 4, 6, 7, 10
p. 712 # 3, 4, 6, 13
Polynomials Test
↳ tomorrow
Perms Quiz → Friday