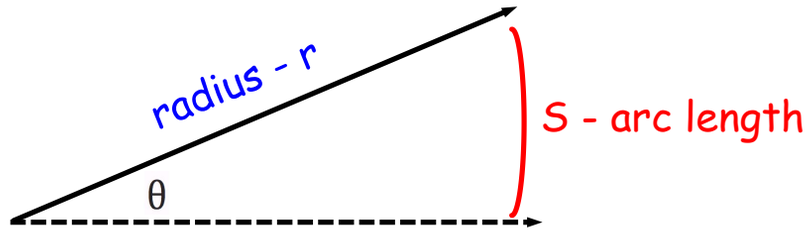
Measurement of Angles

2 Systems

1) **Degree** - unit symbol is "o"
   - ex. 35^{o} vs 35
2) **Radian** - has no unit symbol

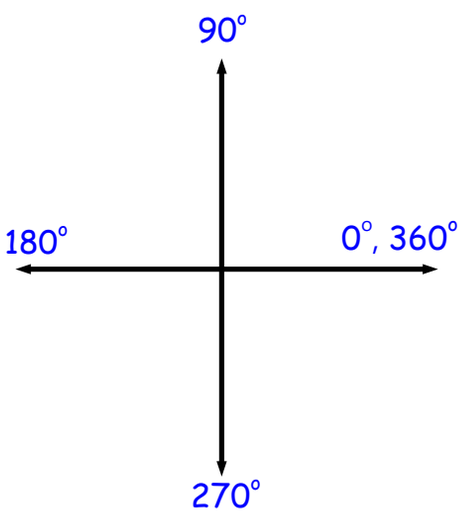


θ (in radians) = $\frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$

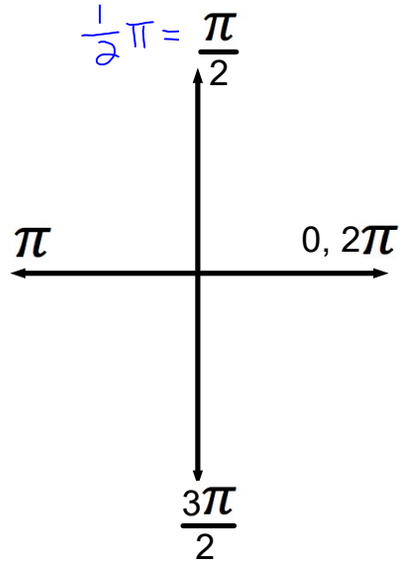
$$\theta = \frac{s}{r}$$

The central angle is subtended or "covered" by an arc length (for a given radius)

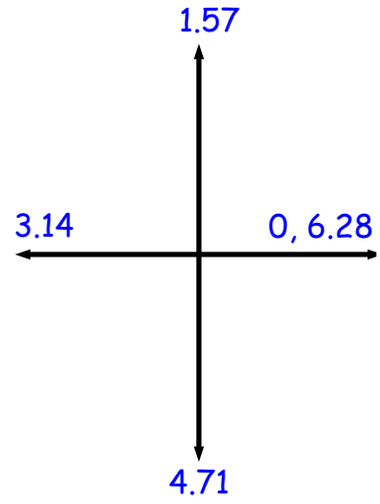
The Connection -> Degrees to Rads



Degrees



Radians
(exact values)



Radians
(Approx.)

Converting Degrees ⟷ Radians

$2\pi = 360^\circ$

$$\frac{x}{\pi} = \frac{x}{180^\circ}$$

ex 1) How many radians is:

a) 30° $\quad \frac{x}{\pi} = \frac{30}{180}$

$x = \frac{30}{180}\pi$

$x = \frac{\pi}{6}$

b) 80° $\quad x = \frac{80\pi}{180}$

$= \frac{4\pi}{9}$

c) 225° $\quad x = \frac{225\pi}{180}$

$= \frac{45\pi}{36}$

$= \frac{5\pi}{4}$

ex 2) How many degrees is:

a) $\frac{3\pi}{5}$ $\quad \frac{\frac{3\pi}{5}}{\frac{\pi}{1}} = \frac{x}{180}$

$(180)\frac{3\pi}{5}\cdot\frac{1}{\pi} = \frac{x}{180}(180)$

$108^\circ = x$

b) $\frac{7\pi}{2}$ $\quad \frac{7\pi}{2}\frac{(180)}{\pi} = x$

$630^\circ = x$

c) $\frac{2}{\pi}$ $\quad \frac{2}{\pi}\left(\frac{180}{\pi}\right) = x$

$\left(\frac{360}{\pi^2}\right)^\circ = x$

$36.476^\circ = x$

Word Problems Radians

$\theta = \frac{s}{r}$

1) Find θ in radians if the arc length is 1.8 cm (s) and radius is 1.2 cm (r).

$$\theta = \frac{1.8}{1.2}$$
$$= 1.5$$

2) A circle has a radius of 3.6 m (r). A string of 6π m (s) is stretched along the circumference. What is the central angle subtended by the arc in degrees.

$$\theta = \frac{6\pi}{3.6}$$
$$= 5.236$$

$$\frac{\frac{6\pi}{3.6}}{\pi} = \frac{x}{180}$$

$$\frac{6\pi}{3.6} \cdot \frac{180}{\pi} = x$$

$$300^\circ = x$$

3) a) Calculate the length of a radius, if the length of the arc is 16 cm and the central angle is 80°.

s

θ must be in radians

Convert $80^\circ \rightarrow$ radians

$$\frac{\theta}{\pi} = \frac{80}{180}$$

$$\theta = \frac{80\pi}{180}$$

$$\theta = \frac{4\pi}{9}$$

$$\theta = \frac{s}{r} \rightarrow r = \frac{s}{\theta}$$

$$r = \frac{16}{\frac{4\pi}{9}}$$

$$r = 11.459 \text{ cm}$$

b) What is the area of the circle? $A = \pi r^2$

$$A = \pi\left(\frac{16}{\frac{4\pi}{9}}\right)^2$$

$$= 412.529 \text{ cm}^2$$

4) An ant is crawling around the edge of a frisbee and gets stuck. Find the distance the ant has travelled if the diameter is 14 inches and the central angle is 60°.

$r = 7$

$\theta = \frac{s}{r}$

$\boxed{s = \theta r}$

$\theta = 60^\circ \times \frac{\pi}{180}$

$= \frac{\pi}{3}$

$s = \frac{\pi}{3}(7)$

$= 7.330 \text{ in}$

Yellow Conversion WS
+ Word Problems

Conversion Word Problems KEY

1. a) 6.052 cm
   b) 115.080 cm^2
2. $\frac{7\pi}{3}$ in
3. 7 times
   14π
4. 35.343 in
5. 125.217°
6. 11.781 ft
7. 26.271°
8. 12.566 in^2
   13.070 in^2
   Pizza by George's
9. 0.524 in/min
10. a) 13.352 ft
    b) 113.490 ft^2