Characteristics of Polynomial Functions

Recall, standard form of a polynomial:

$$\boxed{f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0}$$

where n is a whole number
x is a variable
the coefficients a_n to a_0 are real numbers

The **degree** of a polynomial is the highest power of the variable in the equation.

The **leading coefficient** is the coefficient of the highest power.

Ex. $-2x^3 + 2x^2 + 3$

Deg 3
L.C. −2

Ex. $x^2 + x - 5x^4 - 7x^3 + 6$

Deg 4
L.C. −5

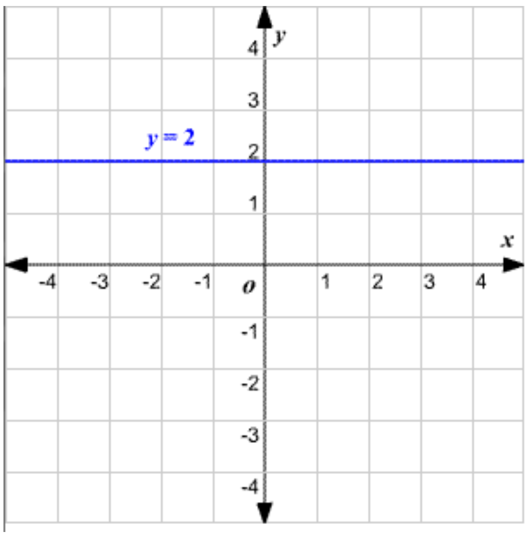
<u>Types of polynomials</u>:
- Linear (degree 1)
- Quadratic (degree 2)
- Cubic (degree 3)
- Quartic (degree 4)
- Quintic (degree 5)

The graph of a polynomial is <u>smooth and continuous</u>
- no sharp corners and can be drawn without lifting a pencil off a piece of paper

Polynomials can be described by their degree:
- Odd-degree polynomials (1, 3, 5, etc.)
- Even-degree polynomials (2, 4, etc.)

Can also have a degree of 0..

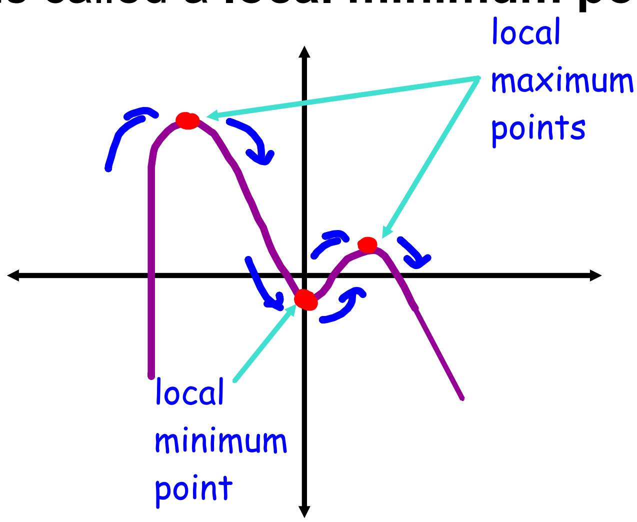
Constant function



$y = 2x^0$

A point where the graph changes from increasing to decreasing is called a **local maximum point**.

A point where the graph changes from decreasing to increasing is called a **local minimum point**.



A graph of a polynomial function of degree n can have at most n x-intercepts and at most (n - 1) local maximum or minimum points

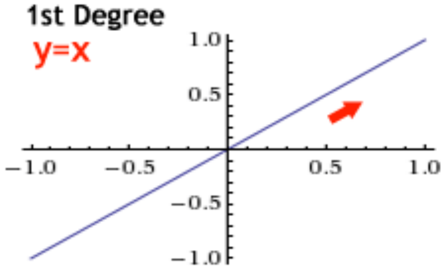
Functions with Odd Powers

Positive	Negative

Positive

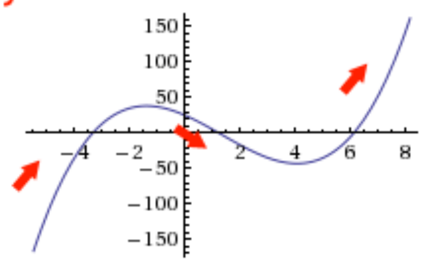
1st Degree

$y=x$



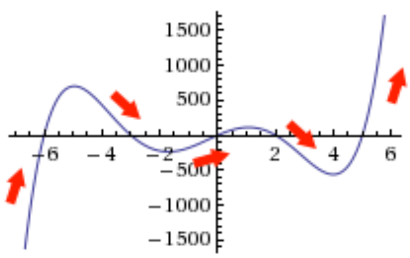
3rd Degree

$y=ax^3+bx^2+cx+d$



5th Degree

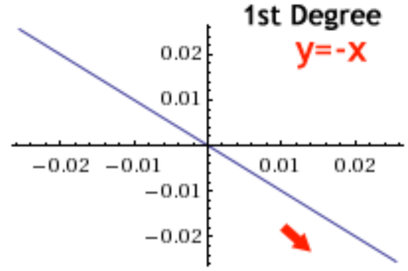
$y=ax^5+bx^3+cx^2+dx+e$



Negative

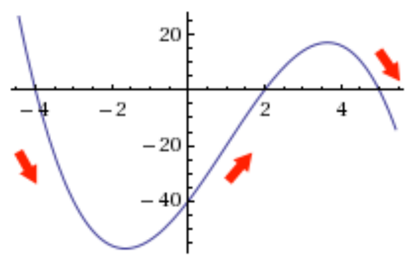
1st Degree

$y=-x$



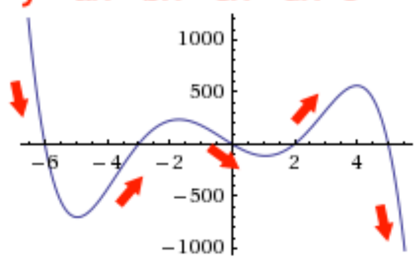
3rd Degree

$y=-ax^3+bx^2+cx+d$



5th Degree

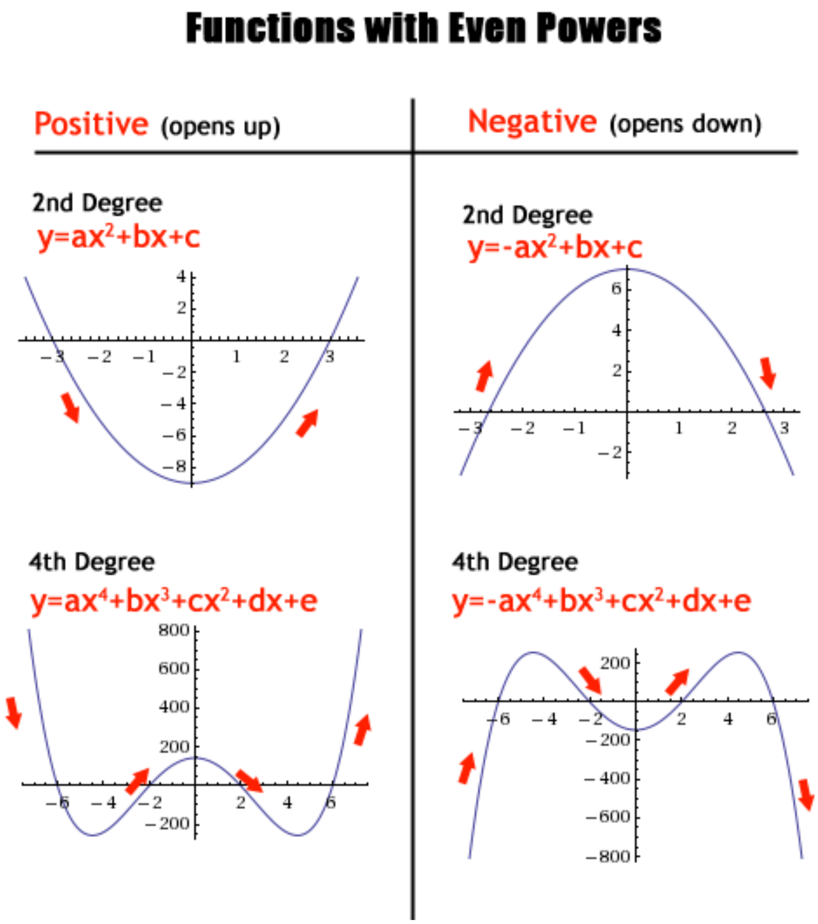
$y=-ax^5+bx^3+cx^2+dx+e$



End behaviour:

<u>Positive, odd</u>:
falls to the left at -∞
rises to the right at +∞

<u>Negative, odd</u>:
rises to the left at -∞
falls to the right at +∞



End behaviour:

Positive, even:
rises to the left at -∞
rises to the right at +∞

Negative, even:
falls to the left at -∞
falls to the right at +∞

Polynomial Matching

What to look for?
- degree
- leading coefficient
- even or odd
- number of x-intercepts
- number of local max/min
- end behaviour

#1.
$f(x) = \frac{5}{6}(x+1)^2(x-1)(x-4)$

#2.
$f(x) = x^4 - 2x^2 + 1$

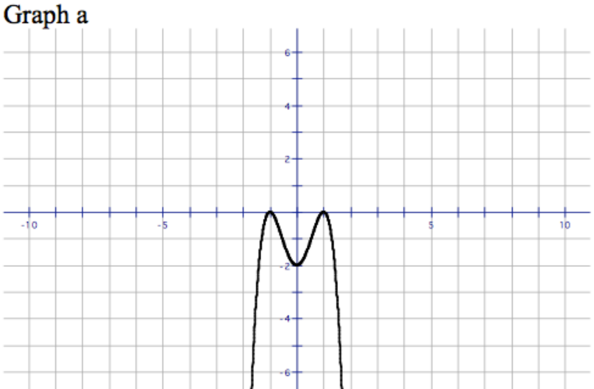
#3.
$f(x) = -3x^5 + 2x^2 - 7x + 1$

#4.
$f(x) = x^3 - 5x$

#5.
$f(x) = -2x^4 + 4x^2 - 2$

#6.
$f(x) = x^5 - 2x^2 + 4$

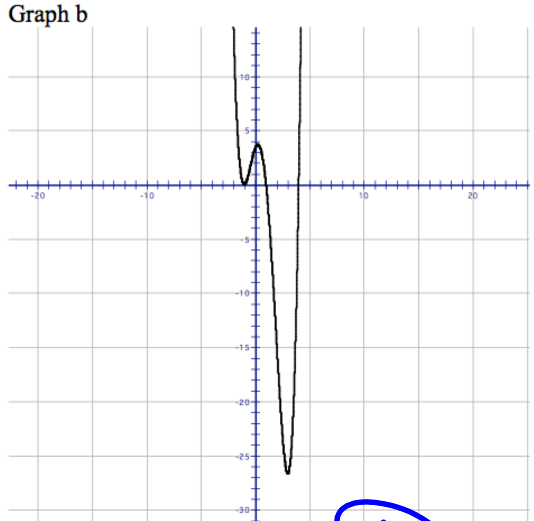
Graph a



L.C. −
Deg 4
y-int −2
x-ints −1, 1

#5

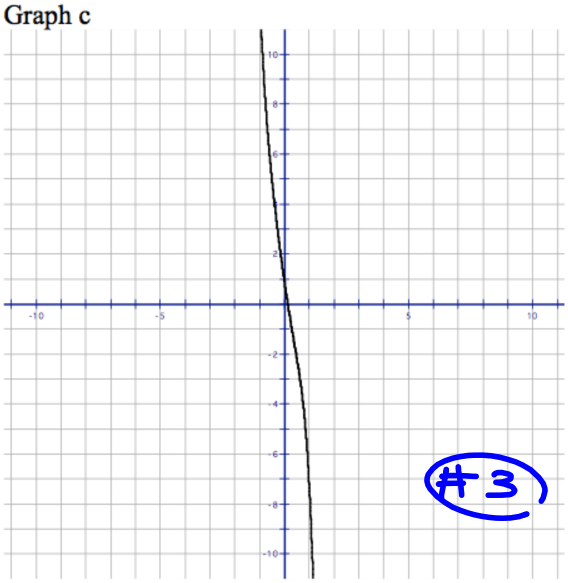
Graph b



Deg 4
L.C. +
x-ints −1, 1, 4
y-int 4

#1

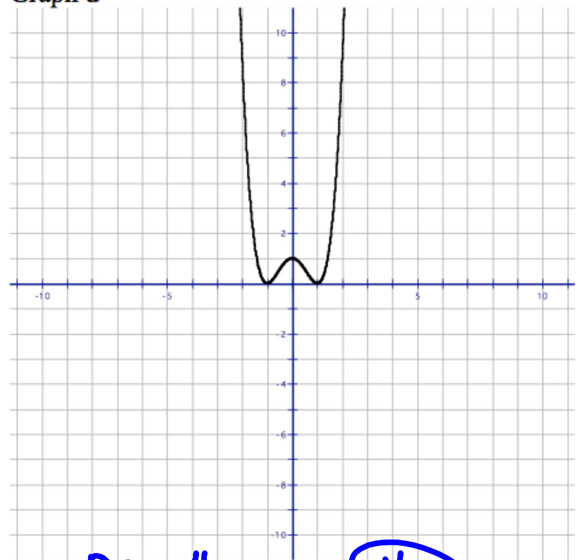
Graph c



#3

LC −

Graph d



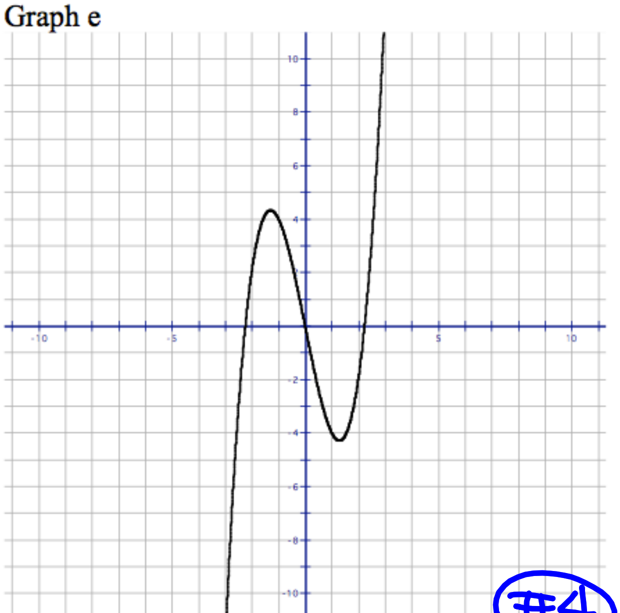
Deg 4

y-int +1

LC +

#2

Graph e



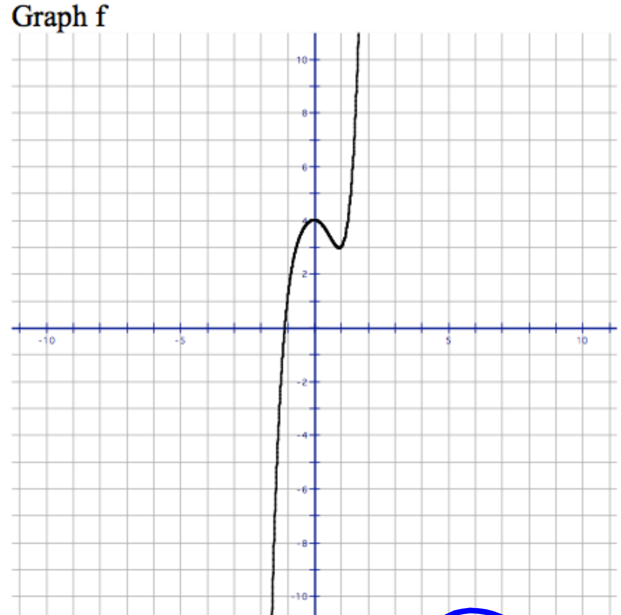
#4

Deg 3 LC +

x-ints y-int

$x \pm 2, 0$ 0

Graph f



LC +

#6

p.46 #1, 3-5, 8

p.54 #1, 2