

Combining Functions

Addition:

$$\begin{aligned}h(x) &= (f + g)x \\h(x) &= f(x) + g(x)\end{aligned}$$

Subtraction:

$$\begin{aligned}h(x) &= (f - g)x \\h(x) &= f(x) - g(x)\end{aligned}$$

Multiplication:

$$\begin{aligned}h(x) &= (f \cdot g)(x) \\h(x) &= f(x)g(x)\end{aligned}$$

Division:

$$\begin{aligned}h(x) &= \left(\frac{f}{g}\right)(x) \\h(x) &= \frac{f(x)}{g(x)}, \quad g(x) \neq 0\end{aligned}$$

Ex 1) If $f(x) = 2x + 3$ and $g(x) = x^2 - x - 5$,
determine $h(x) = (f + g)x$ and $k(x) = (g - f)x$

$$\begin{aligned}h(x) &= f(x) + g(x) \\&= 2x + 3 + x^2 - x - 5 \\&= x^2 + x - 2\end{aligned}$$

$$\begin{aligned}k(x) &= g(x) - f(x) \\&= x^2 - x - 5 - (2x + 3) \\&= x^2 - 3x - 8\end{aligned}$$

Ex 2) Given $f(x) = -x^2 + 3$ and $g(x) = -2(x+1)$, determine
 $h(x) = (g + f)x$ and $k(x) = (f - g)x$.

$$\begin{aligned}h(x) &= g(x) + f(x) \\&= -2(x+1) + (-x^2 + 3) \\&= -x^2 - 2x + 1\end{aligned}$$

$$\begin{aligned}k(x) &= f(x) - g(x) \\&= -x^2 + 3 - (-2(x+1)) \\&= -x^2 + 2x + 5\end{aligned}$$

Ex 3) Given $f(x) = x^2 - 5$ and $g(x) = 3x - 4$, determine $h(x) = (f \cdot g)(x)$.

$$\begin{aligned} h(x) &= f(x)g(x) \\ &= (x^2 - 5)(3x - 4) \\ &= 3x^3 - 4x^2 - 15x + 20 \end{aligned}$$

Note: When $f(x)$ and $g(x)$ are added, subtracted or multiplied, the domain of the new function will be what is common to both functions.

Ex 4) a) Given, $f(x) = \sqrt{x+1}$ and $g(x) = |x|$, determine the domain of $m(x) = (f \cdot g)(x)$.

domain

$$\begin{array}{l} f(x) \quad x+1 \geq 0 \\ \quad \quad x \geq -1 \\ g(x) \quad x \in \mathbb{R} \end{array}$$

$$m(x) = (\sqrt{x+1})(|x|)$$

domain $m(x)$

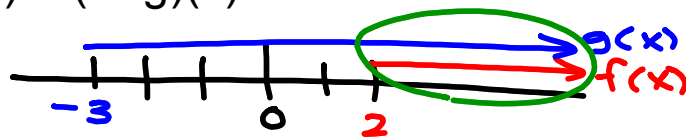
$$\boxed{x \geq -1}$$

b) Given, $f(x) = \sqrt{x - 2}$ and $g(x) = \sqrt{x + 3}$, determine the domain of $m(x) = (f \cdot g)(x)$.

domain

$f(x)$
 $x \geq 2$

$g(x)$
 $x \geq -3$



$$m(x) = (\sqrt{x-2})(\sqrt{x+3})$$

domain $m(x)$ $x \geq 2$

Ex 5) Given $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$,

a) determine the equation of $h(x) = \left(\frac{g}{f}\right)(x)$

b) determine the equation of $k(x) = \left(\frac{f}{g}\right)(x)$

a) $h(x) = \frac{g(x)}{f(x)}$

$$= \frac{2x+6}{x^2+x-6}$$

Factor

$$= \frac{2(x+3)}{(x+3)(x-2)}$$

$$= \frac{2}{x-2} \quad x \neq 2, -3$$

b) $k(x) = \frac{(x-2)(x+3)}{2(x+3)}$

$$= \frac{x-2}{2}, \quad x \neq -3$$

$$= \frac{1}{2}x - \frac{1}{2}$$

Ex 6) Given $p(x) = x^3 - 3$,

Write explicit equations for $f(x)$ and $g(x)$ so that:

a) $p(x) = f(x) + g(x)$

$$\begin{array}{ll} \text{a)} & f(x) = x^3 \quad f(x) = x^3 - x \\ & g(x) = -3 \quad g(x) = x - 3 \end{array}$$

b) $p(x) = g(x) - f(x)$

$$\begin{array}{l} \text{b)} \quad g(x) = x^3 \\ \quad \quad f(x) = 3 \end{array}$$

c) $p(x) = f(x)g(x)$

$$\begin{array}{l} \text{c)} \quad f(x) = x^3 - 3 \\ \quad \quad g(x) = 1 \end{array}$$

d) $p(x) = \frac{f(x)}{g(x)}$

$$\begin{array}{ll} \text{d)} & f(x) = x^3 - 3 \quad f(x) = x^4 - 3x \\ & g(x) = 1 \quad \quad g(x) = x \end{array}$$