Sum + Difference Identities

There are six identities that can be used when an exact value is not on the unit circle:

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Tips:

- Be care to watch the signs.
- Alpha and beta are used instead of theta to differentiate them.
- Your final answer must be a single fraction with one denominator.
- Radicals are okay and do not need to be rationalized.

ex. Determine the **exact value** of $\sin\left(\frac{7\pi}{12}\right)$.

Need to find a pair of exact values that add or subtract to make this.

All these different combination will work and will give you the same final answer. Pick one to use.

$\frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$\frac{9\pi}{12} - \frac{2\pi}{12} = \frac{3\pi}{4} - \frac{\pi}{6}$

$\frac{10\pi}{12} - \frac{3\pi}{12} = \frac{5\pi}{6} - \frac{\pi}{4}$

α β

$$\sin\left(\frac{7\pi}{12}\right)$$

$$= \sin\left(\underbrace{\frac{\pi}{3}}_{\alpha} + \underbrace{\frac{\pi}{4}}_{\beta}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

Plug in values from the unit circle.

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

Multiply and make a common denominator.

ex. Determine the exact value of $\cot(75^\circ)$.

α	β
$30^\circ + 45^\circ$	
$120^\circ - 45^\circ$	
$135^\circ - 60^\circ$	

Must find tan first because there is no formula for the reciprocal functions.

Again, pick any pair of exact values that adds or subtracts to 75°.

$\tan(\alpha - \beta)$

$\tan(75^\circ) = \tan(\underset{\alpha}{120^\circ} - \underset{\beta}{45^\circ})$

$$= \frac{\tan(120^\circ) - \tan(45^\circ)}{1 + \tan(120^\circ)\tan(45^\circ)}$$

$$= \frac{(-\sqrt{3}) - (1)}{1 + (-\sqrt{3})(1)}$$

Plug in values from the unit circle.

$$= \frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$$

$$\cot(75^\circ) = \frac{1 - \sqrt{3}}{-\sqrt{3} - 1}$$

Take the reciprocal of tan to find cot.