Synthetic Division

Synthetic division is a method that only uses the coefficients and constants in a question and is quicker than long division.

Ex 1) $(x^3 - 8x^2 + 5x + 2) \div (x - 2)$

Long division

$$
\begin{array}{r|l}
 & x^2 - 6x - 7 \\
\hline
x-2 & x^3 - 8x^2 + 5x + 2 \\
 & x^3 - 2x^2 \\
\hline
 & -6x^2 + 5x \\
 & -6x^2 + 12x \\
\hline
 & -7x + 2 \\
 & -7x + 14 \\
\hline
 & \underbrace{-12}_{R}
\end{array}
$$

Synthetic division

$$
\begin{array}{r|rrrr}
2 & 1 & -8 & 5 & 2 \\
 & \downarrow & 2 & -12 & -14 \\
\hline
 & 1 & -6 & -7 & \underbrace{-12}_{R}
\end{array}
$$

$x^2 - 6x - 7$

Ex 2) $(2x^4 - x + 3x^3 - 5) \div (x + 2)$

Dividend　　　　Divisor

Steps for Synthetic Division:

Step 1: Write out the coefficients of the dividend (Descending power order) and fill any missing power places with zero

Step 2: To the left place the opposite number of the divisor

Step 3: Bring down the first coefficient

Step 4: Multiply by the divisor and add to the next coefficient

Step 5: Repeat until finished

Step 6: Beginning with the first number, write it with the variable that is one degree less than the dividend. The last number is the remainder.

$$
\begin{array}{r|rrrrr}
-2 & 2 & 3 & 0 & -1 & -5 \\
 & \downarrow & -4 & 2 & -4 & 10 \\
\hline
 & 2 & -1 & 2 & -5 & \underbrace{5}_{R}
\end{array}
$$

$2x^3 - x^2 + 2x - 5$

Ex 3) $(x^3 + x - 4x^2 + 9) \div (x + 1)$

$$\begin{array}{r|rrrr}
-1 & 1 & -4 & 1 & 9 \\
 & \downarrow & -1 & 5 & -6 \\
\hline
 & 1 & -5 & 6 & \underbrace{3}_{R}
\end{array}$$

$x^2 - 5x + 6$

Remainder

$$(x^3 + x - 4x^2 + 9) \div (x + 1) = x^2 - 5x + 6 + \frac{3}{(x+1)}$$

Dividend divisor quotient

Ex 4) $(2x^3 + 3x^2 + 15 - 4x) \div (x + 3)$

$$\begin{array}{r|rrrr}
-3 & 2 & 3 & -4 & 15 \\
 & \downarrow & -6 & 9 & -15 \\
\hline
 & 2 & -3 & 5 & \underbrace{0}_{R}
\end{array}$$

$2x^2 - 3x + 5$

Ex 5) $P(x) = (x^3 - 7x + 6) \div (x - 1)$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \underbrace{0}_{R} \end{array}$$

$x^2 + x - 6$

$$\frac{P(x)}{(x-1)} = x^2 + x - 6$$